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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

THEORY OF LIMIT DESIGN

BY J. A. VAN DEN BROEK,¹ M. AM. SOC. C. E.

SYNOPSIS

This paper shows that the capacity load which a redundant structure can carry is not limited to the load that stresses one member to the elastic limit. In fact, the capacity load of a structure is reached only after as many members of a structure, corresponding in number to the structure's redundants, have all reached their elastic or buckling limit strength. As a load is gradually applied to a structure, its redundant members, in general, are successively stressed, one after another, until they reach their elastic or buckling limit strength. The remaining members may be designated as the primary system which still behaves elastically, and which can carry a still greater load, safely, until one of its members is likewise stressed to the elastic or buckling limit strength. This final load condition is defined as the "capacity load" upon which the design of the structure is based. Thus, if a structure is to have a factor of safety of two, the specified loading is doubled and the designer would find the "limit design" that would permit the redundant members in the structure to reach their elastic limit, or buckling strength. If so loaded some slight permanent set and some residual stresses might result, but the deformations of the structure would still be of the order of magnitude of elastic deformations. The structure could carry half the limit load safely, without any member being stressed to the elastic limit.

INTRODUCTION

Traditionally all theory pertaining to the strength of materials and to structural design is based on two major assumptions: (1) That the material is elastic; and (2) that the principle of superposition applies. To be sure, exceptional problems, to which the principle of superposition does not apply, are treated by successive approximations, but even then the theory of elasticity is followed. In the common problem of designing rivet connections another

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by April 15, 1939.

¹ Prof. of Eng. Mechanics, Univ. of Michigan, Ann Arbor, Mich.

assumption is introduced; namely, that the stresses are distributed equally over all rivets. Very little is said about that assumption other than that it represents established engineering practice. Recently, considerable attention has been paid to various theories of failure. However, the emphasis is placed upon the theory of the failure of materials rather than upon the theory of the failure of structures. The emphasis always has been, and continues to be, directed toward the analysis of stresses.

The man engaged in studies by photo-elasticity sees nothing but strains and talks about nothing but stress. If the theory of columns reveals anything, it is the critical, limit, load of the column, but designers are so much the creatures of habit that they must divide this load by area in order to get back again to talking about stresses. When a structural engineer adopts what is known as "good structural steel," he writes his specifications mostly with reference to stresses, and he expresses his theory in terms of stresses. Practically, however, no bridge or steel frame ever breaks; failure or collapse is the result of excessive deformation. To be sure, in a heap of wreckage it may be possible to find a ruptured steel bar; but there is not one chance in a million that this ruptured bar was the cause of the failure. Structural engineers wax enthusiastic about compound stresses, maximum stresses, working stresses, and critical stresses; in fact, they talk so much about stresses that it would seem that, if they only knew enough about them, they would not need to inquire into anything else. In this connection it is of interest to refer to an experiment² conducted in 1912 in which bars were broken as the result of excessive elongations. In every

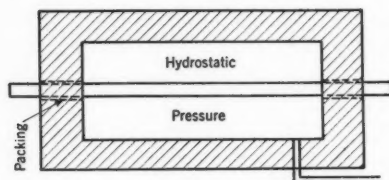


FIG. 1

respect, the break resembled a tensile fracture with no stresses acting in the plane of fracture at the time of rupture. If any stresses were acting on this plane they were compression stresses as a result of friction in the packing, created as the bar was squeezed through (see Fig. 1).

This emphasis on stresses may have been acceptable in the past when civil engineering structures were statically determinate. In such structures each bar, each member, and each cross-section has a definite task to perform. There being no other member or restraint present to relieve conditions, the passing of the elastic limit stress means that excessive deformations of the order of magnitude of ductile deformations (say, from 5% to 10% of the length of the member) will take place without a corresponding increase in the carrying capacity of the structure; and that therefore the usefulness of the structure is brought to an end. Even in that case the excessive deformation is the primary, and the accompanying stress the secondary, consideration. So long as there is a direct relation between the two no practical harm results. However, as long as the engineer continues to talk about stresses when he means deformations, he violates sound engineering philosophy. In the consideration of compression members the logic is essentially the same as that applied to tension members.

² "Breaking Tests under Hydrostatic Pressure and Conditions of Rupture," by P. W. Bridgman, *Philosophical Magazine*, July, 1912.

When a compression member is loaded to its capacity buckling load, although the deformations may continue for a time to be within the elastic range, excessive deformations take place, marking the limit capacity of the structure of which the column is a part.

In the study of statically indeterminate, redundant, structures it is proposed to continue to rely on the criterion that always was the primary one, the criterion of permissible deformations; the derived, or secondary, consideration (namely, the stresses involved) is relegated to its relative position of importance; it is treated as a matter of interest, as a matter that might well help to clarify questions, but not as a matter of primary importance.

A theory is of value in direct proportion to the extent to which it covers the pertinent factors that affect the problem to which it is applied. Any important factor not included within the scope of a theory limits that theory. The theory of elasticity does not give even a hint as to why a perfectly elastic material—a material that is elastic only—is unfit for use as a structural material. The application of the theory of elasticity is thus seriously circumscribed by rules of engineering practice and by stringent specifications. While thus restricted it is open to criticism as a sound and all-inclusive philosophy of structural design.³

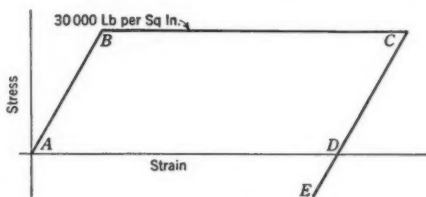


FIG. 2

Structural steel, mild steel, machine steel, open-hearth steel, low carbon steel, and iron, are the various names for the same substance. It possesses an extraordinary property graphically represented by Line BC of the partial stress-strain curve shown in Fig. 2. This property is designated by the engineering term "ductility," which is defined herein as the property of a material by virtue of which it behaves elastically over a considerable range of stress or strain, the strain being proportional to stress, and *vice versa*. Beyond this elastic range the material can suffer considerable deformation without corresponding changes in stress. During these changes in form within the ductile range, the stresses remain essentially constant. It is important to realize that, after passing the elastic limit, the stresses are neither appreciably increased or decreased until very considerable deformation has occurred.

Although the curve of Fig. 2 may appear idealized, this is not necessarily the case to any great degree. To be sure, stress-strain curves frequently are recorded with a bulge instead of the sharp angle shown in Fig. 2 at Point B; or, they appear with a mild transition curve between the two straight lines meeting at Point B. It must be noted that the things tested are specimens and not filaments. For example, an ordinary steel bar suffers a differential stress between material in the outside and the center of the bar due to the cooling of the core (on the cooling bed) after the outside of the bar is already black and hardened. Furthermore, the rate of running the testing machine affects the curve. In other words, variations from stress-strain curves of mild steel, as

³ "Elastic Energy Theory," by J. A. Van den Broek, John Wiley & Son, 1931, p. 231.

shown in Fig. 2, may frequently be assigned to the test specimen rather than to the material. In several hundred tests reported in 1918⁴ the writer found the horizontal part of the curve very pronounced and coincident with the proportional limit. Metallographically, an extraordinary thing happens when the elastic limit is exceeded and the ductile range begins, because it is unusual as judged from the behavior of other material. When the elastic limit is exceeded the crystal grains fail. However, in this connection failure is to be sharply distinguished from rupture. A cleavage is formed within the grain and the one part slips over the other. All this time, however, the two parts of the crystal grain cling to each other tenaciously as if connected by a very viscous glue. When a specimen of structural steel is tested in tension and strained beyond the elastic limit, to the extent indicated by Point C, Fig. 2, and subsequently examined, it is likely that the crystal grains in a part of the specimen will be found still intact, whereas in the other part they will be shown to have suffered permanent deformation.

In the training of engineers, the emphasis on the theory of elasticity leads to a respect for the elastic limit which sometimes borders on fear. To be sure, to exceed the elastic limit in a structure may mean disaster, but inherently it means no such thing. Only when several redundants, functioning to the same end, have each reached the elastic limit, or buckling strength, is failure imminent. Every inch of every wire in every strand of each cable of the Brooklyn Bridge in Brooklyn, N. Y., was strained cold, much beyond its elastic limit, before it was assembled and built into the bridge. This structure has stood up for more than 60 yr and is daily loaded well beyond what it was originally expected to carry. Cold-drawing, cold-rolling, and cold-stamping are processes involving the passing of the elastic limit of the material, sometimes resorted to in order to obtain a desirable shape, but more often done because of their beneficial effects on the properties of the material.

The theory of limit design is the instrument by means of which the capacity strength of a structure is computed. Its philosophy is predicated on the fact that a structure will fail only when as many members, or restraints, as the structure has redundants plus one, are loaded to their capacity strength. From this limit strength, the safe strength of the structure is derived by the introduction of an over-load factor or factor of safety.

It is proposed to present the theory by means of illustrative examples: Four examples in redundant beams referred to in order as Cases 1, 2, 3, and 4, and one example in redundant steel towers.

THEORY

In the theory of limit design the emphasis is shifted from the concept "stress" to that of "permissible deformation." In this connection three dicta of primary importance are proposed:

- (1) A statically indeterminate, redundant, structure is one in which two or more members, reactions or restraints function to the same end;

⁴"The Effect of Cold Working on the Elastic Properties of Steel," Carnegie Scholarship Memoirs, Iron and Steel Inst., Vol. IX, 1918; see, also, *Engineering*, July, 1918.

(2) Sharp distinction is made between deformations of the order of magnitude of elastic deformations and deformations of the order of magnitude of ductile, or of buckling, deformations; and,

(3) When on a well designed, and especially a properly detailed, n -fold redundant structure, n members are stressed to their elastic limit, or up to their critical buckling load, the deformations involved are of the order of magnitude of elastic deformations until an $(n + 1)$ th member has reached its elastic limit or its critical buckling capacity.

Case 1.—To find the maximum uniformly distributed load, applied in one direction and sense, that a beam built in at both ends can carry with a factor of safety of 2.—The elastic and ductile properties of the material of which the beam is made are assumed to be as represented graphically by Fig. 2. The beam is 6 ft long, fully restrained at both ends, and of rectangular cross-section, 6 in. \times 2 in., as shown in Fig. 3.

The section modulus of this beam is $\frac{b h^2}{6} = \frac{2 \times 6^2}{6} = 12$. The maximum resisting moment that the beam can develop is:

$$M = \frac{s I}{c} \dots \dots \dots (1)$$

or, $M = 30\,000 \times 12 \text{ in-lb} = 30\,000 \text{ ft-lb}$.

The elastic behavior of this beam is represented by Fig. 3. From the theory of static equilibrium, the increment of bending moment between the end and middle of the beam is $\frac{w L^2}{8}$. The theory of elasticity is necessary, however, to tell that the ratio between M_1 and M_2 is as 1 : 2, or, in other words, that the bending moment at each end is $\frac{w L^2}{12}$, whereas that in the middle is $\frac{w L^2}{24}$.

The elastic curve presents essentially two cantilever beams, $A-B$ and $C-D$, and one simple beam, $B-C$.

To express the foregoing argument in physical terms, the beam fully restrained at both ends presents a case of two cantilever beams and one simple beam working together to the same end—that of carrying a uniformly distributed load w over the entire length of the beam. The combined task of the simple beam and the two cantilever beams is obvious. The theory of elasticity, however, is necessary to determine just how this task is distributed in simple beam action and cantilever beam action.

If no part of the beam is to be stressed beyond the elastic limit, the capacity load w that the beam can carry is $w = \frac{30\,000 \times 12}{6 \times 6} = 10\,000 \text{ lb per ft}$; and the safe load, with a factor of safety equal to 2, is 5 000 lb per ft.

It is of interest to inquire into what would happen if this beam were subjected to a load greater than 10 000 lb per ft. As soon as this load is exceeded, the elastic limit in the outer fiber in a section at the wall is also exceeded. The steel at this point, which is ductile, flows somewhat in a manner indicated by Line $B-C$ in Fig. 2. The bending moment undergoes a change as indicated by the change from the dash line to the solid line in Fig. 4. The points of inflec-

tion, B and C , shift to Points B' and C' respectively (Fig. 4), and this process continues until the bending moment in the center has reached its critical value (equal to that at the wall) namely, 30 000 ft-lb, and the corresponding load on the beam has reached its critical value, w' lb per ft (Fig. 4). If loading beyond w' lb per ft is attempted, it appears that ductile deformation at both center and end of the beam continues unhindered. The deformations assume proportions of the order of magnitude of ductile deformations, and the end of the beam's usefulness is reached.

It has been stated that the theory of elasticity was necessary in order to determine the true ratio between M_1 and M_2 within the elastic range. In this instance, having determined by the foregoing argument that the capacity

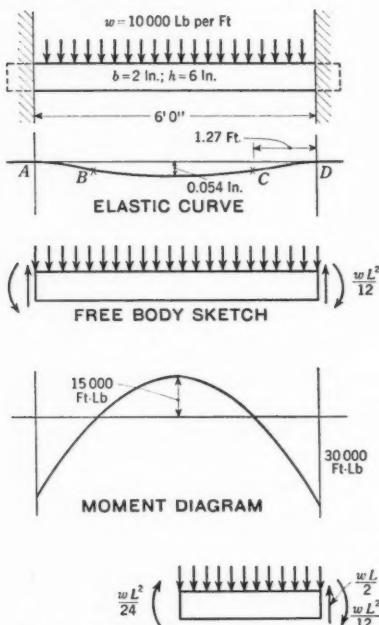


FIG. 3.—BUILT-IN BEAM LOADED TO JUST WITHIN THE ELASTIC LIMIT

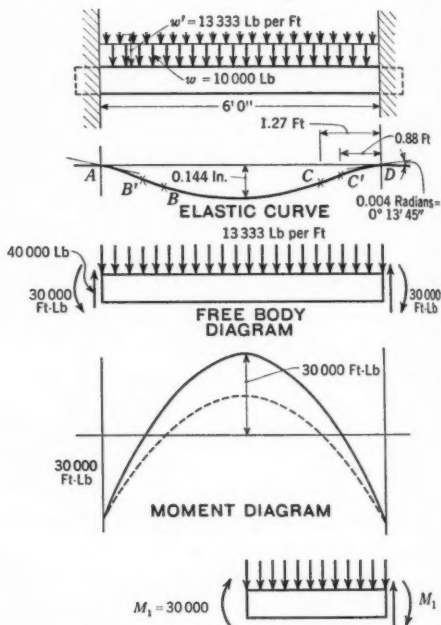


FIG. 4.—BUILT-IN BEAM LOADED WITH ITS CAPACITY LOAD

moments at the center and the end of the beam must be equal before disastrous results will take place, one may proceed with the arguments of static equilibrium to obtain the capacity carrying load w' : $w' \frac{L}{2} \times \frac{L}{4} = 2 M_1 = 60\,000$ ft-lb; and, $w' = 13\,333$ lb per ft.

The application of the factor of safety again is contrary to the usual practice. Instead of applying it first in order to determine working stresses, it is applied last in order to determine safe load. Thus, if the capacity load is 13 333 lb per ft, and the factor of safety is 2, the safe load is 6 667 lb per ft. This change in the order of applying the factor of safety is not the trivial matter it may seem. By applying the factor of safety immediately, the design r finds the working

stress in the middle of the elastic range and is helpless in extricating himself from the difficulty. By reserving the application of the factor of safety to the last and basing his analysis upon the elastic limit stress, he is in a position to study the effects of a slight ductile flow. For working loads, the ultimate safe load will fall back well within the elastic range. This means that ductile flow will most probably never occur in the life of the structure. It would occur only in case of a 50% over-load, and even then, if a slight permanent deformation is permissible, such permanent deformation would still be of the order of magnitude of elastic deformation up to a 100% over-load.

It may be argued that when a built-in beam is loaded to full ductile capacity, the magnitude of the bending moment at the walls would be somewhat in excess of that of the bending moment at the center. This merely means that the capacity load w' is slightly in excess of the computed 13 333 lb per ft. Thus, by ignoring it and by assuming the moment at the wall to remain at the constant magnitude of 30 000 ft-lb, the argument is on the side of safety. At this stage the writer intentionally omits studying this question in detail so as to avoid spending his efforts on the study of secondary considerations and thus diverting attention from the essential arguments.

When all the fibers in one cross-section of a beam are stressed to the elastic limit, the stress-distribution diagrams over the cross-section would appear as two rectangles, and not as two triangles. In that case the resisting moment is expressed by the formula:

$$M = 2 s_1 \bar{y} A \dots \dots \dots (2)$$

in which s_1 is the elastic limit stress; A is the area on one side of the neutral axis; and \bar{y} is the distance to the centroid of this area. Thus, the value of the limiting resisting moment becomes greater than that given by the usual formula, Equation (1). When all fibers along one section of the beam have reached the elastic limit this value would be 50% greater in a beam with a rectangular cross-section and 6.67% greater in a C.B. 12 in. by 12 in. by 65 lb beam.⁵

Case 2.—To find the maximum uniformly distributed load, applied an indefinite number of times in one direction and in one sense.—When a ductile steel is strained beyond the elastic limit ($B-C$, Fig. 2) and then the straining effect removed, the return path of the stress-strain curve ($C-D$, Fig. 2) is essentially parallel to the initial path $A-B$; that is, the material again behaves according to Hooke's law. If care is not taken at this point, one can easily become so lost in details that essentials are overlooked. As to whether or not Curve $C-D$, Fig. 2, is parallel to Line $A-B$ depends primarily on the time that is allowed to elapse between the moment the over-strain is completed and the moment the straining effect is removed.⁴ Furthermore, when the stress is completely released, and then reversed, the material continues to behave elastically as indicated by Line $D-E$, Fig. 2. To assume Line $C-E$ in Fig. 2 to be a straight line is not exactly true, but by assuming conditions that would prevail ultimately if sufficient time elapses, conclusions are reached which, in certain cases, argue against the adoption of the theory of limit design (see Case 3). These conclusions, however, are on the safe side.

⁵ See a dissertation, by E. O. Scott; presented to the University of Michigan in 1939, in partial fulfillment of the degree of Doctor of Philosophy.

If the load of 13 333 lb per ft, in Case 1, Fig. 3, is removed, the beam behaves elastically. The stresses return toward zero and in certain points may pass through zero and reverse. The elastic behavior of a beam under a negative load of 13 333 lb per ft is represented by Fig. 5. The final condition of the beam under a positive load of 13 333 lb per ft is represented by Fig. 4. When the free-body sketch of Fig. 5 is superimposed on the free-body sketch of Fig. 4, the result is a free-body sketch of the beam with the load removed (see Fig. 6). When the bending moment of Fig. 5 is superimposed on the bending moment of Fig. 4, the result is as demonstrated in Fig. 6, which shows the residual bending moment left in the beam after it had been loaded to full-capacity ductile load and after this load had been removed.

The conclusions to be drawn from the foregoing appear interesting and important. What would happen if a structure is loaded beyond the elastic limit? This question is introduced not because it is actually proposed ever, consciously, to permit it, but to find a better engineering philosophy than the

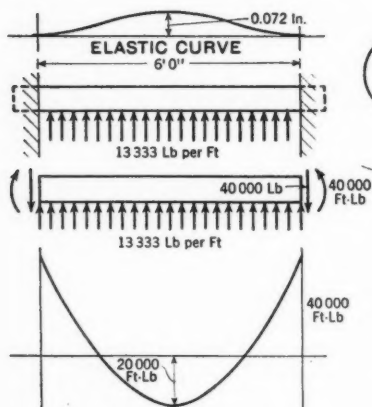


FIG. 5.—RESTRAINED BEAM LOADED ELASTICALLY IN A NEGATIVE SENSE

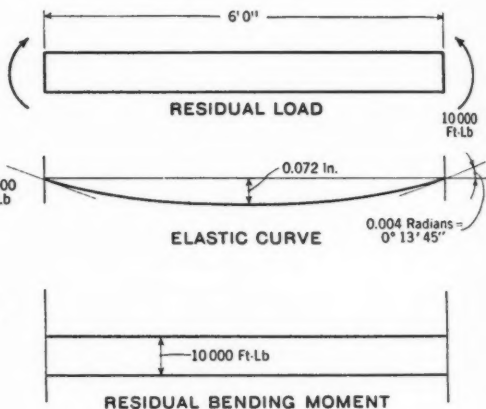


FIG. 6.—BUILT-IN BEAM AFTER REMOVAL OF LIMIT LOAD

one based on the consideration of working stresses. It must be kept in mind that after the capacity load is found the safe load is derived from it, and stipulated to be a safe margin below the capacity load. It is seen that a capacity ductile load induces deformations of the order of magnitude of elastic deformations. A removal of the capacity load leaves the structure with a slight permanent set, and, what is more important, with a residual stress condition set up in it. If the full capacity load is again applied in the same direction and sense, the behavior is completely elastic. For example, under completely elastic behavior the bending moment of a beam, loaded as shown in Fig. 7, develops bending moments of 20 000 ft-lb at the center and 40 000 ft-lb at the wall. When this loading and these moments are superimposed upon the residual loads and moments, represented by Fig. 6, the resultant loads and moments are those shown in Fig. 4, this time, however, obtained elastically. This means that for an application of one type of limit loading, up to ductile capacity, the

first application and removal of the load induce in the structure residual stresses of such nature and magnitude that subsequent application of loads does not call forth additional ductile deformation. After the first application of the loads the behavior of the structure thenceforth again becomes completely elastic.

Case 3.—To find the maximum uniformly distributed load applied an indefinite number of times in one direction, but in alternating sense (complete reversal of load), which the beam can carry with a factor of safety of 2.—Beginning with the residual stress condition of Fig. 6: A load of 6 667 lb per ft is applied in the upward direction, uniformly distributed. This induces a moment of 20 000 ft-lb at each end, which is the maximum load that the beam can carry within the elastic range. The imposition of a 20 000 ft-lb of bending moment at the wall, added to the already existing moment of 10 000 ft-lb, would bring the stress up to the elastic limit at that point (see Fig. 8). At the same time the bending moment in the center of the beam would be reduced to zero. The condition discussed in Case 1 may be repeated; that is, the beam may be loaded with an upward load of 13 333 lb per ft until the capacity bending moments of 30 000 ft-lb are developed in the center as well as at the ends. This, however, takes place at the expense of additional ductile yield, a ductile yield greater in amount than that involved in Case 1, because the stress-differential is greater. At the end of such loading, if the load is removed, the beam is left again with a residual stress, this time in a sense opposite to that shown in Fig. 6. If, next, a downward load is applied, this process repeats itself. Each time, however, additional ductile strain is involved. Each strain may be of the order of magnitude of elastic strains. A relatively few repetitions, however, will make the addition of these strains objectionable and dangerous. This phenomenon is not to be likened to what is commonly understood by fatigue. This is not a question of thousands or millions of repetitions of load but rather a question of, perhaps, a dozen repetitions or so. What happens may more aptly be likened to a wire nail held in a vise and

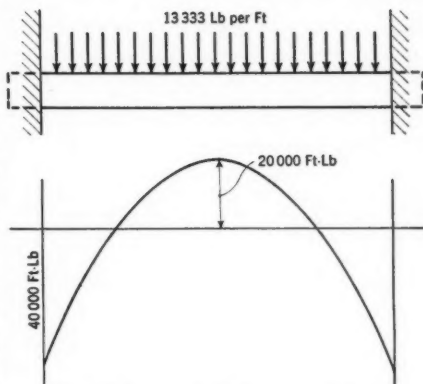


FIG. 7.—RESTRAINED BEAM LOADED ELASTICALLY IN A POSITIVE SENSE

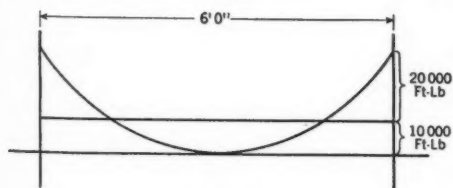


FIG. 8

bent back and forth, each time assuming a permanent set until, in comparatively short time, it breaks.

If the foregoing arguments are applied to a reversal of load, smaller than the capacity ductile load but greater than the most severe load within the

elastic range (say, a reversal of a load of 11 000 lb per ft), then it develops that each application of the load in opposite sense leaves residual stresses of a sense favorable to a load such as that last applied, but unfavorable to a load of a sense opposite to that last applied. The permanent sets would be smaller in magnitude, but would nevertheless be accumulative. There seems no other alternative, then, but to conclude that, if a complete reversal of stress is possible, the elastic limit is again the final criterion of strength. The saving feature of this argument lies in the circumstance that, in bridges and in steel frames, partial reversal of stress is comparatively rare, and complete reversal of stress is so rare as to be almost non-existent. (To avoid disastrous results under a complete reversal of loading the sum of the positive and negative loadings must not exceed an amount equal to twice the maximum elastic loading. Thus the beam in this example will sustain an indefinite application of an 11 000 lb per ft positive load and a 9 000 lb per ft negative load, or a 12 000 lb per ft positive and an 8 000 lb per ft negative loading.)

This phenomenon of residual stresses and accumulation of ductile deformations is of primary importance in the theory of limit design. Accumulation of ductile deformation may occur under moving loads even when no reversal of loads takes place.

Case 4.—To find the maximum (limit) value of two concentrated loads, P_1 , spaced 10 ft apart, that can be moved an indefinite number of times along a beam of two spans, each 10 ft long, and of section modulus, $\frac{I}{c}$, equal to 30.—

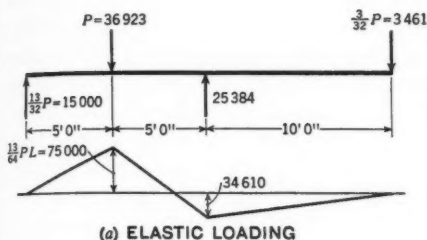
It is proposed to discuss this case in four steps:

- (a) Find the single limit load P_1 applied in the center of the left span, and the residual moments and reactions resulting from that load;
- (b) Find the double limit loads P_2 applied in the center of each span simultaneously, and the residual moments and reactions resulting from them;
- (c) Find the limit loads P_3 that may be applied alternately: Singly in the center of one span, or simultaneously in pairs, at the center of both spans; and,
- (d) Find the limit loads P_4 that may be moved along the beam an indefinite number of times.

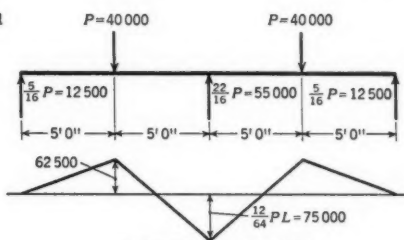
Step (a).—Fig. 9(a) shows the capacity load, concentrated in the center of one span, that the beam can carry without exceeding the elastic limit; Fig. 9(b) shows the limit load P_1 concentrated in the center of one span; and, Fig. 9(c) shows a negative loading of magnitude equal to P_1 when the beam behaves altogether elastically, the loads and reactions being in direct proportion to those of Fig. 9(a). As previously stated, if the limit load P_1 (Fig. 9(b)) is removed, even if the maximum concentrated loading within the elastic range is 36 923, the effect is the same as if a negative elastic loading (Fig. 9(c)) is superimposed upon the loading shown in Fig. 9(b). The material then behaves elastically as represented by Line CDE (Fig. 2). Fig. 9(d) represents the residual reactions and moments when the loading in Fig. 9(c) is superimposed upon the loading in Fig. 9(b); that is, when Load P_1 of Fig. 9(b) is removed.

As a check of the loading and moment shown in Fig. 9(b), reverse the loading represented in Fig. 9(c) and superimpose it upon the beam in Fig. 9(d). This is a case similar to the one discussed in detail under Case 2.

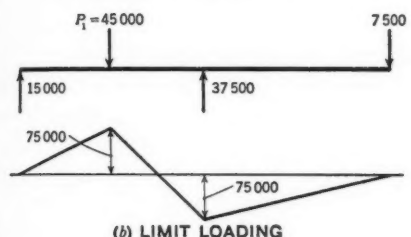
Step (b).—The arguments for Step (b) on the basis of Fig. 10 are similar to those previously advanced in connection with the representations of Step (a)



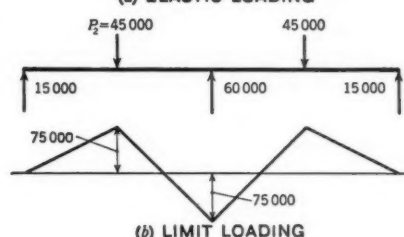
(a) ELASTIC LOADING



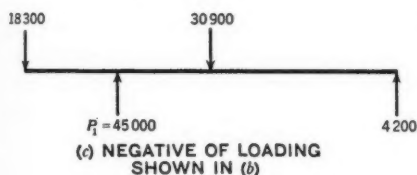
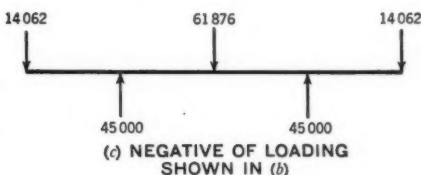
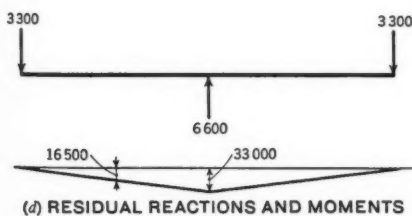
(a) ELASTIC LOADING



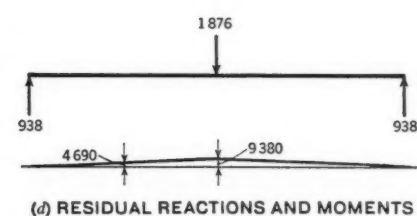
(b) LIMIT LOADING



(b) LIMIT LOADING

(c) NEGATIVE OF LOADING
SHOWN IN (b)(c) NEGATIVE OF LOADING
SHOWN IN (b)

(d) RESIDUAL REACTIONS AND MOMENTS



(d) RESIDUAL REACTIONS AND MOMENTS

FIG. 9

FIG. 10

on basis of Fig. 9. The capacity limit loads P_1 and P_2 , in each instance, are 45 000 lb. After they were once applied they could each be applied an indefinite number of times and the behavior of the structure would be perfectly elastic with each subsequent application of the loading.

Step (c).—However, if the loading P_2 represented by Fig. 10(b) is applied after the loading P_1 (Fig. 9(b)) had been previously applied and removed, it

would meet with a residual loading condition (Fig. 9(d)) which is responsible for an initial bending moment over the middle support of 33 000 ft-lb. Thus the elastic limit over the middle support is reached sooner than it is in the case represented by Fig. 10(b), in which the loading presumably started from a zero base line. This would therefore result in ductile flow of somewhat greater intensity than that noted in the case given in Fig. 10(b). If subsequently the P_2 -loading is removed and the P_1 -loading is again applied, a similar situation results. As the P_1 -loading is applied it meets with an initial positive bending moment of 4 690 ft-lb at the middle of the left span. This in turn requires more ductile flow before the full limit load of P_1 (Fig. 9(b)) can be carried. Each successive application of a P_2 -loading, after a P_1 -loading has first been applied and removed, is seen to require additional ductile flow which, in turn, would spell disaster.

The value of P_3 , the limit load which can be applied singly at the center of one span, in pairs at the center of both spans, and applied alternately an indefinite number of times, may be computed as follows:

It will be recalled that, so far as repetitions of limiting loads are concerned, in accordance with the theory of limit design (Case 2), the structure is left with a residual load in a pre-stressed state, as the result of over-strain. This residual loading may be beneficial, if not too large, and if not of the wrong sign. In the case of repetitions of the same load (Case 2) the residual stresses are always the maximum and of the correct sign, thus making it possible for subsequent limit loading to take place entirely within the elastic range. When there is complete reversal of limit loading (Case 3) the residual loading is of the wrong sign, thus making the theory of limit design inapplicable for practical purposes. In the case of alternate loading (the loading shown in Fig. 9(b), alternating with the loading shown in Fig. 10(b)) the residual loading may well be too large. This does not mean, however, that a smaller residual loading could not be beneficial.

The largest possible values for the residual loading can be computed so that, with a design based upon that value, the beam will be stressed alternately to full capacity—first stressed to full capacity at the center of the span, and then stressed to full capacity over the middle support, without, however, resulting in further ductile flow, or over-strain. From the study of Fig. 9(a) the maximum moment in the center of the span is known to be $\frac{13}{64} P L$. Fig. 10(a) shows that

the maximum moment over the middle support is $\frac{12}{64} P L$. The ideal residual

bending moment would be the dash line in Fig. 11, so defined that the ordinate, measured from the dash line as a base to the point marking the maximum moment in the center of the span, shall equal the ordinate measured to the point marking the maximum moment over the middle support. If the distance to the vertex of the residual bending moment diagram is represented by x ,

$$\frac{13}{64} P_3 L - \frac{x}{2} = \frac{12}{64} P_3 L + x \dots \dots \dots (3)$$

and, $x = \frac{1}{96} P_3 L$. The limiting bending moment is 75 000 ft-lb; and, therefore, $\frac{12}{64} P_3 L + \frac{1}{96} P_3 L = \frac{19}{96} P_3 L = 75\,000$ ft-lb; or, $P_3 = 37\,895$ lb. The answer is given in five significant figures, not because the writer is of the opinion such accuracy is justified in this instance, but because it may be of interest, if any one should be so inclined, to substitute the value of 37 895 lb for the 36 923 lb in Fig. 9 and compute the residual reactions and moments. Then repeat the argument of Step (c) with the value 37 895 lb substituted for 45 000 lb. It will be found that the alternate loading with this value of P_3 remains exactly within the elastic range.

In the case of moving loads, a similar procedure may be followed.⁶ Suppose a single load, P_4 , rolls along the beam. The maximum negative bending moment will be at the middle support. It will occur when Load P is at the distance $0.42265 L$ from the middle support and when it is of the magnitude $0.096225 P L$. The maximum positive moment in both spans of the beam is expressed by the equation:

$$M = (2 L^3 x + L^2 x^2 - 4 L x^3 + x^4) \frac{P}{4 L^3} \dots \dots \dots (4)$$

x being measured positive to the left from the middle support. The value of the maximum negative moment as well as the graph giving the maximum positive moment is represented in Fig. 12.

If it is assumed that the magnitude of P_4 is very small to begin with and that it becomes progressively larger with each passage along the beam, the elastic limit in positive bending will be reached and a residual stress built up favorable to the application of loads of the same magnitude subsequently applied. If the load P_4 is quite large the resulting over-strain will cause the residual loading, although favorable to the positive moments, to become unfavorable to the negative moment over the support. This is explained in Case 4, Step (c). The most favorable residual bending moment is the one represented by the dash line in Fig. 12. It may be found graphically by drawing the dash line so that the maximum positive ordinate shall equal the maximum negative ordinate. For the purpose of this paper the solution has been found analytically instead of graphically. It is found that the maximum moment equals $0.1744 P_4 L$. Equating this moment to 75 000 ft-lb and solving for P_4 , as was explained in Case 4, Step (c): $P_4 = 43\,000$ lb. It is to be noted that the maximum positive ordinate, measured from the dash line,

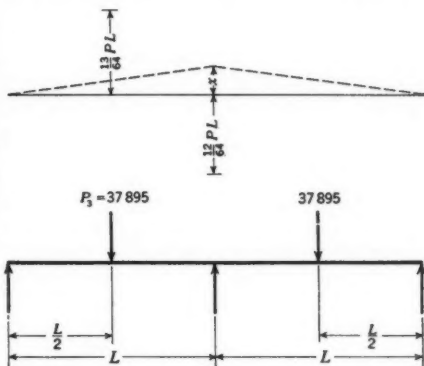


FIG. 11

⁶ Über die Bemessung Statisch unbestimmter Stahltragwerke unter Berücksichtigung des Elastisch-Plastischen Verhaltens des Baustoffes, by Hans Bleich, *Bauingenieur*, 1932. Heft 19/20, p. 261.

does not occur in the center of the span. The physical meaning is as follows: When a load of 43 000 lb passes over the beam for the first time, it causes overstrain and builds up residual reactions and moments. These residual reactions and moments are of a magnitude such that, during subsequent passages of the load P_4 (43 000 lb), the beam behaves entirely elastically. The elastic limit stress will be reached at both the point of maximum positive bending and that of maximum negative bending; but this elastic limit stress is not exceeded. The safe load, on the basis of a safety factor of two, would thus be one-half of 43 000 lb, or 21 500 lb.

Step (d).—Suppose that a number of loads, each of magnitude P_5 , and 10 ft apart, roll along the beam. As the loads first come on, only one span will be loaded with a single load. The curve for maximum positive moment (Fig. 13), therefore, will be identical with the curve shown in Fig. 12. The maximum negative moment, however, will occur when the load P_5 is in the center of each span. This maximum negative moment will be $0.1875 P_5 L$.

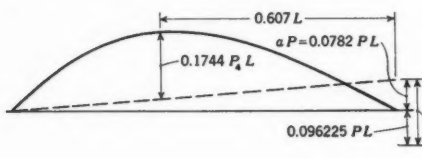


Fig. 12

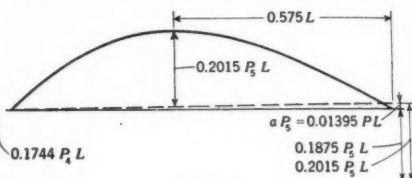


Fig. 13

The residual bending moment that would permit perfectly elastic behavior for an indefinite number of load applications, without resulting in additional over-strain, is again shown by the dash line. An analytical solution gives the value for the equal positive and negative moments as $0.2015 P_5 L$. The value of P_5 , which would build up this ideal residual moment, is obtained from the equation, $0.2015 P_5 L = 75\,000$ ft-lb; or, $P_5 = 37\,231$ lb.

Fig. 14 shows the results of the foregoing discussion arranged in a manner convenient for easy comparison. Fig. 14(b) shows the maximum loads the beam can carry without any part of the beam being stressed beyond the elastic limit. Fig. 14(a) shows the limit loads the beam can carry under similar conditions of loading and in accordance with the theory of limit design. The first application of the load would build up residual stresses that would thereafter insure perfect elastic behavior, provided the limit loads are never exceeded. Sketches (1), (2), (3), and (4), Fig. 14, represent loads applied only at the points shown; and, Sketches (5), (6), (7), and (8), Fig. 14, represent moving loads.

DESIGN OF A STEEL TOWER

Fig. 15(a) represents the upper three panels of a steel tower, the general, over-all, dimensions of which suggest a transmission tower. A transverse concentrated capacity load of 60 000 lb is assumed to be applied at Point A at the top of the tower. This load is purposely assumed rather large (about four times the value of a similar load on a representative transmission tower) because it is proposed to design this tower with only readily available data.

It is desirable to avoid the unusually small structural members commonly used in transmission tower design, the properties for which would not be easily available to any one inclined to check this design. Therefore, in this design, all structural members are limited to angle-irons of equal legs. The slenderness

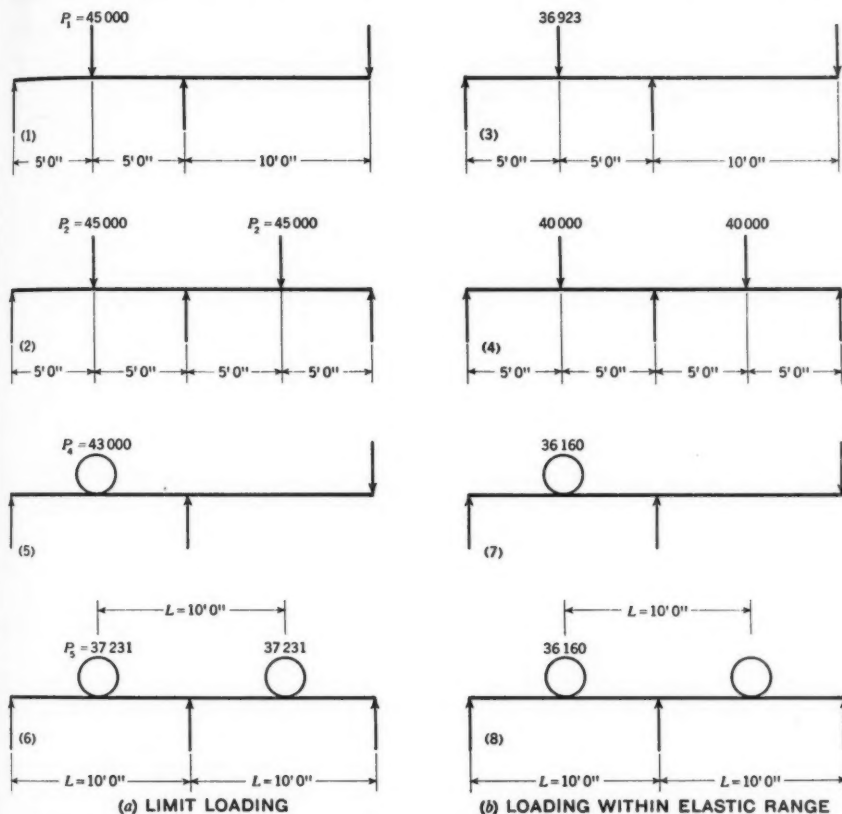


FIG. 14.—ELASTIC LIMIT STRESS IS 30 000 POUNDS PER SQUARE INCH; AND, SECTION MODULUS IS 30

ratio is limited to 200; and the joints are assumed welded instead of riveted, or bolted, so that the question of punched, or reamed, holes is eliminated. This, then, does away with the uncertainty as to how much to reduce the cross-section area in tension due to the presence of the holes. The stiffening effect of the welded joints and the resulting moments are ignored, which is in agreement with practice and is on the side of safety. The buckling strength of the compression members is computed on the basis of Euler's formula,

$$P = \frac{\pi^2 E A}{\left(\frac{L}{r}\right)^2} \dots \dots \dots (5)$$

The tower is to be designed symmetrically with reference to the center line.

It must be kept in mind that the purpose of this paper is to discuss and illustrate the theory of limit design. In listing the foregoing specifications for tower design the writer is guided solely by the one consideration, that his work shall lend itself to convenient and independent checks. It is not to be assumed, therefore, that he means to express a preference for Euler's formula over some other column formula; nor that he wants to be considered as committed to the feasibility of welding transmission towers in the field. The restriction to

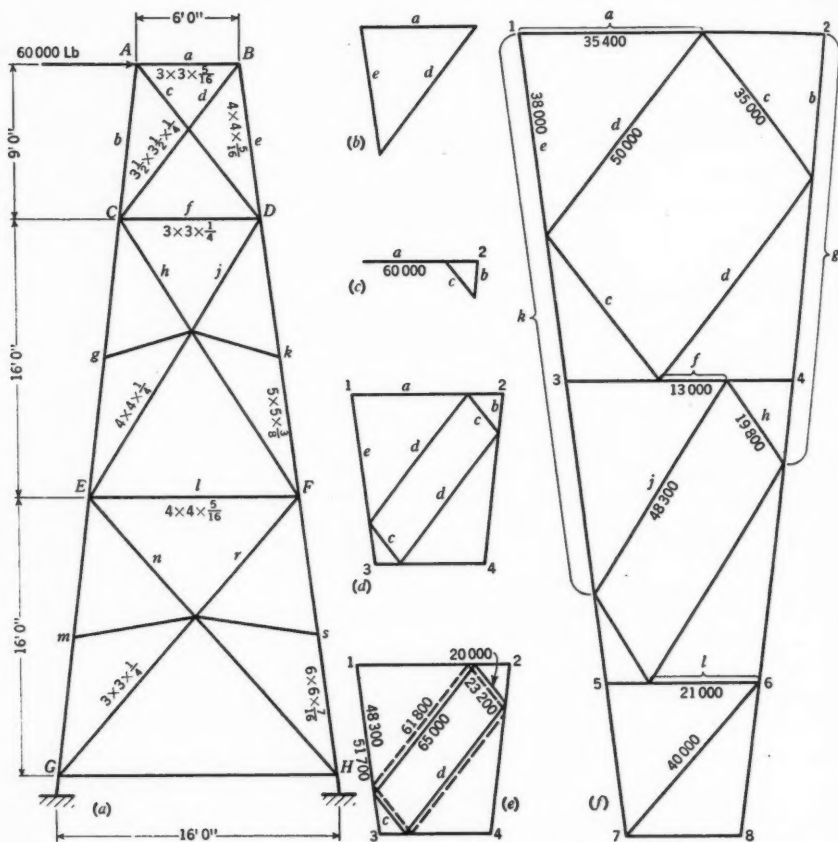


FIG. 15.—DESIGN OF A STEEL TOWER

equal-leg angles is equally arbitrary and would of course have no merit in a practical case.

As indicated near the beginning of Case 1, the sum of the moments at the wall and in the center of the beam is $\frac{w L^2}{8}$. This conclusion was reached on the basis of the theory of static equilibrium. Any other theory applied to the problem is supplementary. On the basis of the theory of elasticity it was found that the ratio of the two moments is 2 : 1. On the basis of the theory

of limit design it was held that, before failure could occur, the ratio between the two moments became 1 : 1.

Similarly, whether the theory of elasticity or the theory of limit design is used in the design of the tower represented by Fig. 15(a), either theory is only supplementary to the theory of static equilibrium. In any case the forces around Joint *A* are in equilibrium and their force diagram must close. The same is true about the forces around Joint *B*. Qualitatively, these respective force diagrams appear as shown in Fig. 15(b) and 15(c). Since F_a , the force in Member *a*, occurs in both force diagrams, the two may be superimposed to appear as in Fig. 15(d). An infinite number of force polygons for the joints at both Points *A* and *B* are possible. It is significant that one of the limiting conditions is to the effect that they have the force F_a in common.

In an analysis on the basis of the theory of elasticity dimensions are assumed for the members of the tower, and the stresses are computed, being careful not to allow the stress in any of the members of the tower to exceed the safe working stress. Furthermore, the designer must also make certain that the computed stresses generally are not unreasonably low. If one or the other of these conditions cannot be satisfied, some member (or members) of the structure is changed and the analysis repeated.

An analysis on the basis of the theory of limit design begins by assuming the dimensions of one member and proportioning the neighboring members in relation to it. The choice of sizes of members is limited by those commercially available, and the result of the theory is limited by the available theory of capacity strength of either the compression or tension members. These two limitations apply to any conceivable theory one might use, and they detract from the theory of limit design no more than they detract from the theory of elasticity. With these two limitations in mind, one might design the tower in the manner of the "one hoss shay," so that, when it collapses, it will collapse by failure of all its component parts simultaneously.

Consider the top panel (*A B C D*, Fig. 15(a)) of the tower. If Strut (*a*) were made extremely light, Triangle *B C D* would become relatively ineffective and the compression member *C* would have to be made extra heavy. On the other hand by making Strut (*a*) heavy and the compression member *C* light, the work of carrying the load would be thrown on Triangle *B C D* and Bar *d* would have to be made heavy. Either procedure is undesirable and is contrary to the specification that the tower is to be designed symmetrically relative to the center line. This calls for equal sizes of Members *c* and *d* and it is desirable that both be loaded as nearly as possible with their maximum safe load.

For Bar *a*, connecting Points *A* and *B* (Fig. 15(a)), select a $3\frac{1}{2}$ in. by $3\frac{1}{2}$ in. by $\frac{1}{4}$ in. angle-iron. Its buckling capacity strength would be (by Equation (6)):

$$\frac{9.87 \times 30\,000\,000 \times 1.69}{104.3^2} = 46\,000 \text{ lb.}$$

It can develop this strength only on condition that the other members of the panel, of which Bar *a* is a part, are capable of developing proportional strength. That is, if (Fig. 15(d)) a force equal to 46 000 lb (see Fig. 15(d)) is laid off to represent the force in Bar *a*, then Bars *c* and *d* must be capable of developing forces at least equal to the magnitudes marked *c* and *d* in the force diagram (Fig. 15(d)).

The effective length of Diagonal c is measured from the point of intersection of the diagonals to Point D . While one of the diagonals is in compression the other is in tension, and since they are supposedly connected at the point of intersection, the tension diagonal restrains the compression diagonal. This effective length of Member c scales 80 in. A 3 in. by 3 in. by $\frac{1}{4}$ in. angle for Diagonal c would develop a capacity buckling strength of 23 200 lb. Since the force vector c (Fig. 15(e)) scales only 20 000 lb this angle would be satisfactory. However, the force vector d (Fig. 15(e)) scales 65 000 lb. The capacity strength of a 3 in. by 3 in. by $\frac{1}{4}$ in. angle in tension is only $1.44 \times 30\,000$ lb = 43 200 lb. This size of angle for the diagonal d would thus be insufficient. A 3 in. by 3 in. by $\frac{3}{8}$ in. angle, with an area of 2.11 sq in., could develop a capacity tensile strength of 63 300 lb. Except for the stipulation of symmetry the use of this angle could be justified as follows: The force vector c could be moved parallel to itself until it reached a value equal to the 23 200-lb capacity strength of Diagonal c , as shown by the dash line in Fig. 15(e) (Diagonal c being 3 in. by 3 in. by $\frac{1}{4}$ in.). In that case it appears that the diagonal d would have to carry only 61 800 lb, and that the 3 in. by 3 in. by $\frac{3}{8}$ in. angle is large enough for that purpose. If there were a greater choice of commercial sizes one could select a still smaller angle for Diagonal d —one capable of developing a capacity tensile strength of only 61 800 lb. If the top panel of the tower had been thus designated, that panel would fail in the two diagonals before failing in Strut a .

Since the designer is restricted by a specification demanding a symmetrical structure he could, of course, also change the diagonal c to an angle, 3 in. by 3 in. by $\frac{3}{8}$ in. This appears wasteful, making some other solution desirable.

If, for Bar a , he selects an angle 3 in. by 3 in. by $\frac{5}{16}$ in., which has a capacity buckling strength of 35 400 lb, he obtains, for the forces around the Joints A and B , the force diagram shown in Fig. 15(f). A $3\frac{1}{2}$ in. by $3\frac{1}{2}$ in. by $\frac{1}{4}$ in. angle iron, instead of Bar c , with an effective column length of 80 in., will develop a buckling strength of 37 200 lb and a capacity tensile strength, instead of Bar d , of 50 700 lb. In each case this is a little more than required. The advantage in changing from a design based on Fig. 15(e) to one based on Fig. 15(f) is greater than the saving in weight of Bars a , c , and d would imply. For the stress distribution in Bars a , c , and d , as shown in Fig. 15(f), the compressive force to be carried by Member e (Fig. 15(f)) scales 38 000 lb. Its effective length is 109 in. A 4 in. by 4 in. by $\frac{5}{16}$ in. angle will just develop a buckling strength equal to 38 000 lb. The same angle in the place of Member b will develop a tensile capacity load equal to 72 000 lb, which is almost double the required capacity. However, the design of Bar e is the controlling consideration.

By strictly analogous processes, a force diagram may be drawn for the entire tower that will satisfy only the conditions of static equilibrium (Fig. 15(f)). Any continuous force diagram, and one that closes, may be regarded as satisfactory. The members of the tower are to be so selected that their capacity strength, either in compression or tension, as the case may be, will be greater than the value of the corresponding force indicated on the force diagram. In that manner the capacity load of the tower is obtained. If the tower is to be designed for an over-load capacity, say, of 100% (in other words, with a

factor of safety of two), begin by multiplying the load for which the tower is to be designed by this over-load factor. Continuing thus, design the structure, and upon completion obtain the safe load by dividing the capacity load by this same over-load factor.

Comment on Steel Tower Design.—A few generalizations at this point would seem to be of interest. Figs. 15(d), 15(e), and 15(f) demonstrate that the forces in Members *c* and *d*, in each case, add up to the same value, 85 000 lb, which is equal to the distance graphically represented by the diagonal 2-3 (Fig. 15(f)). This is a geometric proposition, true for any parallelogram with sides parallel to the diagonal bracing *c* and *d* (Fig. 15(a)) that may be drawn within the isosceles trapezoid 1-2-3-4 (Fig. 15(f)). Taking advantage of this fact, one may scale the distance 4-5 (Fig. 15(f)) and find it equal to 68 300 lb, thus concluding that the diagonal-bracing in Panel C D E F (Fig. 15(a)) must carry this load. Next, select from available commercial stock, the smallest angle-irons, of which the combined capacity strength in tension and compression is at least equal to 68 300 lb. The effective length of the compression member *h* scales 135 in. The capacity buckling strength of a 4 in. by 4 in. by $\frac{1}{4}$ in. angle, 135 in. long, is 19 800 lb. Its capacity strength in tension is 58 200. The combined capacity of the two angles acting in the place of *h* and *j* (Fig. 15(a)), therefore, would be 78 000 lb. This is somewhat in excess of the required value, but, limited as he is by available commercial sizes, it is the closest approximation the designer can achieve. The next step is to lay off the full value of 19 800 lb (Fig. 15(f)), as this has a tendency to reduce the value of the compressive forces in Member *f* as well as in that of Member *k* (Fig. 15(f)).

The individual force polygon for the various members acting at each joint may be found in Fig. 15(f), which shows that the force required to be carried by the diagonal bracing *n* and *r*, in the bottom panel, scales 40 000 lb. The effective length of the compression diagonal *n* scales 144 in. A 3 in. by 3 in. by $\frac{1}{4}$ in. angle would develop a capacity tensile strength of 43 200 lb whereas its slenderness ratio is $\frac{144}{0.59} = 244$. According to the specifications a column with a slenderness ratio greater than 200 is not permitted; or, if used, is not to be considered as carrying any load. The tension diagonal *r*, therefore, is assumed to carry the full required load of 40 000 lb, which it is capable of doing. The functioning of the diagonal bracing in the bottom panel is in agreement with engineering practice as applied to the bracing of the center panel in highway bridges. In these bridges it has generally been the practice to brace the center panel with two light diagonals capable of carrying only tension. While one of the diagonals is loaded in tension the other is assumed to carry no load whatsoever. This is another instance of the theory of limit design being applied in an isolated case.

It may be observed (see Fig. 15(f)) that the higher the value one can assign to the compression members *c* and *h* the smaller will be the value of the forces in the compression members *e*, *f*, *l* and *k*.

If the values of the forces carried in Members *c* and *h* could be stipulated to be equal to those in Members *d* and *g*, respectively, the parallelograms *cd*

and hj would appear as perfect "diamonds." They would be symmetrical to the center line of Fig. 15(f), and the force, Distance f , which now scales 13 000 lb, would be zero. In that case the cross brace f , between Points C and D , could be omitted.

There are two ways in which this object could be accomplished. Members c and h could either be designed heavy enough to carry one-half the diagonal bracing load, or more cross bracing could be introduced, thus reducing the effective lengths of c and h , so that their slenderness ratio would become less than 100. Either of these designs would be more costly than one including the cross braces f and l .

If the foregoing conclusions are compared with those dictated by the theory of elasticity, it is clear that, by no stretch of the imagination, can there be a larger force in Member d than in Member c . In order to avoid complications, consider only the top panel and assume the distance $C-D$ to remain unaltered. Only if Member a were infinitely stiff would Points C and D carry the same load. With an angle-iron of ordinary proportions acting in the place of a , the load in Member c is certain to be larger than the load in Member d . The writer offers no objection to these conclusions; he objects only to making them a basis for design. Without computing exactly what the stress in Member c would be under the theory of elasticity, one needs to conclude only that it would be greater than half the sum of 50 000 lb and 35 000 lb—that is, greater than 42 500 lb. The stipulated angle ($3\frac{1}{2}$ in. by $3\frac{1}{2}$ in. by $\frac{1}{4}$ in.) would not carry this load; and thus, on the basis of the theory of elasticity, the member c would have to be made heavier, and therefore more costly. The theory of limit design takes account of the fact that, as the capacity load is applied to the tower, the member c bends out somewhat as it continues to carry load, but it refuses to carry more than its own capacity load. From the time that the capacity load in c is reached the member d assumes the burden of taking care of the load increments until its capacity strength also is reached. Only then may one look for serious results. Since the tower is supposed to be designed for an over-load capacity of, say, 50% or 100%, the working load permitted is 40 000 lb (or 30 000 lb instead of 60 000 lb) and thus limited this buckling does not occur although it would do no harm if it did.

The theory of elasticity, presumably, defines all stresses within the elastic limit and no stresses in the range of capacity loads. The theory of limit design defines no stresses within the elastic limit and is the only theory that offers some definition of conditions under capacity loads. Since, in reality, it is the capacity load that a structure can carry which should govern as a basis for design, the theory of elasticity fails where the theory of limit design serves as a sound philosophy and guide.

DESIGN OF RIVETED CONNECTIONS

The study of rivet connections is especially important in connection with a proper estimate of the theory of limit design. Consider for example Fig. 16. According to the theory of rivet design, in the space between the two rivets toward the right, the upper plate would carry one-fifth of the load, whereas the

lower plate carries four times as much. It is to be remembered that, although the universal application of the theory of elasticity is seriously challenged, the assertion that a material like steel is almost ideally elastic, up to a certain stress, is not questioned at all. The ratio of 1 : 4 between the stresses in the upper and lower plates is obviously inconsistent with the fact that the distance between rivets in these plates is the same. According to the theory of elasticity the two outer rivets would carry the major part of the load and the three inner ones would practically be "duds." An interesting point is that up to a certain load unquestionably this circumstance would prevail. Nevertheless, the theory of elasticity fails as completely in the design of rivet connections as it does in the analysis of many other redundant structures.

Rivet connections appear to be "giving ground" to welded joints. It may be hoped that they soon will disappear entirely. Nevertheless, they have been designed in vast numbers for many years past and the simple rules of design have been proved very satisfactory. Eminently satisfactory results must be rooted in a satisfactory and sound theory. This theory is the theory of limit design. The pity is that it has been so extensively practiced, yet so little talked or written about. A rivet connection of purely elastic material

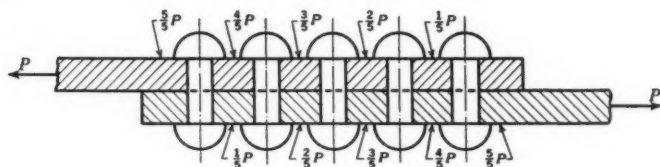


FIG. 16

is as impossible to conceive as the punching of holes into cold glass plates and riveting them together with glass rivets. If the material were purely elastic, and nothing but elastic, the theory of elasticity would apply. Provided a riveted joint, of purely elastic material, such as shown in Fig. 16, could be made, the outer rivets would carry nearly all the load. When they fail the next two inner rivets would take the load over, and finally the center rivet would be stressed with the full load. The entire process no doubt would be completed in a very small fraction of a second; but, the failure would nevertheless be progressive.

The saving feature of rivet connections does not derive from the theory of elasticity, but is found in the most rigid specification which stipulates that the most ductile of steels be used. The philosophy which underlies and justifies the proved practice is the theory of limit design. According to this theory the distribution of stress in rivets is at first in agreement with that given by the theory of elasticity. As the load gradually increases, the outer rivets (Fig. 16) reach their elastic limit. They then yield slightly while they continue to carry a constant load, and only then are additional increments of the loads transmitted to the inner rivets. The structure as a whole will not fail until all rivets have passed their elastic limit. In other words, for ordinary loads, the stresses in rivets may not approach within a thousand per cent of what the theory of design indicates. The point is that the only stresses which really

interest the designer are those under failing loads, and the theory of limit design teaches that for capacity loads, provided the material is ductile, the assumption of equal distribution of loads over all rivets is justified.

It is not to be thought that the problem of rivet connections is discussed at length because it offers an ideal example of the application of the theory of limit design. It is, in fact, a very poor illustration. The theory of limit design presupposes that the first redundant member in a structure that reaches its capacity strength will yield sufficiently, under a constant force, to permit other redundants to be taxed to their full capacity strength before complete failure can take place. The potential yielding of one rivet under capacity load is, after all, limited. The reason the practice in connection with the design of rivet connections is here submitted as a practical example of the application of the theory of limit design is that, for practical purposes, the theory of elasticity fails completely. The very simple rules that have governed rivet design for years have been proved to be reasonably satisfactory. If their use is to be philosophically justified, the arguments derived from the theory of limit design are the only ones that can do so. When it is argued (as is done with renewed emphasis since the appearance of the British Report of the Steel Structures Research Committee of 1931 to 1936⁷) that long rivet connections are to be avoided, this argument, again, follows directly from considerations of the principles of limit design.

PRACTICAL APPLICATION

In 1914 the writer designed a steel trestle for a three-track railway in Canada. The outbreak of the World War delayed the project and when it was reviewed again, other designers revised the plans in reinforced concrete, recommending a structure similar to that of Fig. 15(a) but, naturally, without diagonal bracing. The writer was told that his design of the structure was unquestionably correct, but that it was designed according to no theory at all. In fact, the structure was designed according to the best theory available. The capacity restraining moments at all the corners had been computed and from this the strength of the entire structure was determined. When this strength proved sufficient the design was approved. The interesting psychological point would seem to be the unwillingness to dignify as a theory a philosophy that ran counter to the established theory of elasticity. This attitude still appears to be very common and is giving way but slowly.

Another tale, for the truth of which the writer cannot vouch, but which sounds very plausible, is as follows: A final authority on office building design was asked what he did when a complicated analysis of wind stresses was submitted for his approval. His reply is reported to have been that he locked himself in his office, computed the safe resisting moments at top and bottom of columns and that of their connections to the floor beams in any one story, and compared them with the increment of moment from floor to floor (that is, the total wind load above the floor in question multiplied by the story height). If the former exceeded the latter, he would approve the design. This, to the

⁷ See, for example, Second Rept. of the Steel Structures Research Committee, Dept. of Scientific and Industrial Research; published by His Majesty's Stationery Office, London, England, 1934.

writer, appears to be a sound procedure; but why lock the door? In fact, the writer questions whether there is any theory of wind stresses superior to this one.

The theory of limit design has been applied to riveted connections as long as there have been rivets. The writer was aware of its being applied successfully to reinforced concrete structures on the Canadian Pacific Railway as long ago as 1914. In his own teaching of the elastic energy theory, the writer has always ended the course with a discussion of "The Limitations of the Theory of Elasticity."⁸

With reference to the application of the theory of limit design to towers it is of interest to cite two contradictory viewpoints. In 1932, Hans Bleich⁹ stated that, in order to be effective, all compression members must be so proportioned that the stresses induced (including residual stresses) would never reach the buckling limit. Therefore, according to Bleich, the theory of limit design should never be applied to trusses.

On the other hand, Prof. N. C. Kist has held¹⁰ that "every assumption of statically indeterminate values is correct if the dimensions of the structure are designed accordingly." Furthermore, he is undoubtedly responsible for the statement¹¹ that, "as a rule, the engineer need not apply the theory of elasticity in order to assign values for statically indeterminate quantities leading to an efficient and economical design."

The difference expressed in the foregoing viewpoints is significant. Contrary to Bleich's dictum, the writer has shown that a genuine economy, of time and material, is inherent in the theory of limit design as applied to trusses. The fear of buckling often appears deeply ingrained. This fear is fully justified from the standpoint of the theory of elasticity, or when statically determinate structures are involved. In the analysis of redundant structures a slight buckling (only enough to permit some other detail to function to its full capacity strength) need not be feared.

Professor Kist's dictum sounds sweeping but it is fully justified as long as complete reversals of capacity loads are guarded against.

CONCLUSION

The foregoing examples of the practical application of the principles of limit design are relatively few. It is believed, however, that practices essentially similar to the principles treated in this paper are commonly applied; yet the leaders in the profession, the writers of textbooks on strength of materials and theories of design, to the writer's knowledge, have consistently avoided the subject. In most of the writings on plasticity, even when the writer has ductility in mind, its application to the theory of design is ignored. Those who have written on the subject and shown its relation to problems in design are Kazinecy

⁸ "Elasticity Energy Theory," by J. A. Van den Broek, John Wiley & Sons, Inc., N. Y., 1931.

⁹ *Bauingenieur*, 1932; Heft 19/20; p. 264.

¹⁰ International Cong. for Metallic Structures, Liège, Belgium, 1930.

¹¹ Explanatory notes accompanying Specifications of the Royal Society of Engineers of the Netherlands, 1933, Art. 2, p. 20.

(1914),¹² Kist (1917),¹³ and Bleich (1932), and, in the United States, Edward Godfrey,¹⁴ M. Am. Soc. C. E. A number of papers on the subject appear in the report of the Second Congress of the International Association for Bridge and Structural Engineering, Berlin-München, October, 1936. Notable among these is a paper by H. Maier-Leibnitz.

The dominance of the theory of elasticity has resulted not only in an over-emphasis of the idea of "stress," but is also responsible for the fact that most published test data are given only in so far as stresses and deformations within the elastic limit are concerned. For example, numerous test results are available on columns. Such results give the buckling loads but very little or no information in regard to the load-deformation relationship after the buckling has begun. Since this is the type of information in greatest demand relative to the theory of limit design, a vast field for experimentation would be opened up if this theory should meet with favor.

In closing, it seems only fair to call attention to one more fact: The main part of the foregoing examples has been directed to the study of moving loads and reversal of loads. The exceptions to, and the limitations of, the theory of limit design have been emphasized to an extent that might well create a false perspective. Suppose a rivet connection, designed according to conventional rules, were subjected to reversal of capacity loads. Unquestionably, the connection would fail after a limited number of such full reversals. The point is that, although reversal of loads and stresses occurs, it seldom occurs for capacity loads. So seldom does it occur that the rules of procedure (that is, the application of the simpler rules of the theory of limit design) are either not affected by it, or affected very little.

Suppose a transmission line were to be covered with a heavy coating of sleet and subjected to a gale from the southwest, transverse to the direction of the line. Suppose, then, if supposition can be so fantastic, that the following year the same sleet condition occurred and the gale blew from the northeast. This is where the suppositions begin to appear absurd. Extreme over-load conditions may occur a few times during the life of the structure. That they occur with the same intensity, the same direction and complete reversal of sense, is against all probability. Even so, according to the theory of limit design they could so occur a few times without disastrous results.

It would seem that the structural engineer has more or less consciously designed in accordance with the principles of the theory of limit design, while paying "lip service" to the theory of elasticity. Is it not appropriate to take stock and develop the theory which most closely fits the facts and appears to have the largest range of application where redundant structures are concerned?

¹² *Betonzemle Nos. 4, 5 and 6, 1914, Kisértetek Befalazott Tartokkod; "Bemessung von Statisch Unbestimmten Konstruktionen unter Berücksichtigung der Bleibenden Formänderungen," International Congress for Metallic Structures, 1930; also "Versuche mit Innerlich Statisch Unbestimmten Fachwerken," by Dr. Ing. G. von Kazinczy, Der Bauingenieur, 1938.*

¹³ "Does a Theory of Strength, which Is Based on the Assumption of Proportionality between Stress and Strain Lead to Good Construction of Steel Bridges and Buildings?" by N. C. Kist, Inaugural address, Technical University, Delft, Holland, 1917. Also, "Ductility as a Base for Design—Computation of Steel Bridges and Structures instead of Proportionality of Stress and Strain," by N. C. Kist, International Congress for Metallic Structures, Liège, Belgium, 1930.

¹⁴ "Secondary Stresses in Bridges," by Cecil Vivian Von Abo, Assoc. M. Am. Soc. C. E., *Transactions, Am. Soc. C. E.*, Vol. 89 (1926), p. 193.

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PAPERS

LATERAL SPILLWAY CHANNELS

BY THOMAS R. CAMP,¹ M. AM. SOC. C. E.

SYNOPSIS

The term "lateral spillway channel" is used in this paper to designate an open channel which receives inflow throughout its length, laterally from one or both sides, and discharges the accumulated flow at a point in the channel, usually one end. When used for reservoir spillways, these channels are sometimes referred to as "side channel spillways." On a smaller scale they are found in all water and sewage treatment plants. The best known example is the "wash-water gutter" of rapid filter plants. Other examples are the main wash gutter or "gullet" of a rapid filter, effluent flumes or "launders" of settling basins, and the underdrains of trickling filters. The term "lateral spillway channel" is used herein in order to avoid confusion with "side weirs," with which the flow is in the reverse direction or from the channel over the weir.

In this paper, equations for the water-surface profile are derived for channels of constant width which receive their inflow uniformly throughout their length. The equations hold for both level and sloping invert and for bottoms of any regular shape. A term is included to account for friction, and the equations are put in simple form and are accompanied by a graph to facilitate their use in design. Some experimental measurements are presented for the purpose of verifying the theory, and establishing the approximate value of the friction factor for channels used in water and sewage treatment plants.

INTRODUCTION

A number of attempts have been made to develop equations for the flow in wash-water gutters. Some of these equations were semi-rational, containing arbitrary coefficients. One equation failed to account satisfactorily for a sloping invert. Another equation was based upon the assumption that the water surface draw-down is equal to the velocity head. It has since been

NOTE.—Written comments are invited for immediate publication; to insure publication, the last discussion should be submitted by April 15, 1939.

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shown,² and will be made evident in this paper, that the draw-down is much greater than the velocity head, even if friction is neglected.

Julian Hinds,² M. Am. Soc. C. E., in working with spillways for dams, was probably the first to make a substantially correct statement of the fundamental differential equation for the water-surface profile of lateral spillway channels. Mr. Hinds' theory is based upon the momentum principle. The differential equation is for the case in which the increment of inflow per unit of channel length is constant; and it is applicable to channels of any shape. The momentum of the inflowing water and friction in the channel are neglected in Mr. Hinds' equation. It is not readily integrable for the general case, and solutions are obtained by successive approximations dealing with short lengths of the channel. Friction losses are computed separately.

Mr. Hinds' differential equation was amended by Mr. Henry Favre³ to include a friction term and to account for a component of inflow velocity in the direction of the axis of the channel. Mr. Favre integrated the equation for short lengths of the channel in which average values of the cross-sectional area and hydraulic radius could be used without introducing appreciable error. Solutions by means of Mr. Favre's equation require successive approximations with short channel lengths, as is the case with Mr. Hinds' equation, but the computations are probably expedited because of the presence in the equation of a term for friction.

For channels of variable or irregular shape, such as are required for the economical design of dam spillways, a general solution probably necessitates the use of the Hinds or the Favre method. In water and sewage treatment plants, however, lateral spillway channels are usually of constant width, with inverts having no slope, or a constant slope. For such structures a simplification of the equations and methods is both possible and desirable.

The differential equation developed in this paper is identical with that used by Mr. Favre, except that the friction term includes the well-known Weisbach-Darcy friction factor. Because of the simplicity of the channel shapes considered, it is possible to integrate the equation approximately so that it may be used for the entire channel without serious error. For accurate solutions, and for the determination of several points on the water-surface curve, successive approximations are required; but the procedure is much simplified.

FUNDAMENTAL DIFFERENTIAL EQUATION

For a steady state, the flow of water in a lateral spillway channel is not only non-uniform but is complicated by the fact that both velocity and discharge increase from zero at the up-stream end to maximum values at the down-stream end. In the following development, a constant increment of discharge is assumed per unit length of channel, and the momentum of the inflow is neglected. For a changing velocity, the momentum principle states that the accelerating force is equal to the time rate of change of linear momentum; or, that is, to the mass of water flowing times the acceleration produced.

² *Transactions, Am. Soc. C. E.*, Vol. 89 (1926), p. 885.

³ *Engineering News-Record*, October 25, 1934, p. 520.

For a rectangular flume of width b (Fig. 1), consider two vertical cross-sections at Sections m and n , a distance Δx apart. The momentum at Section m , for the mass per second, is

$$M_m = \frac{Q w V}{g} \dots \dots \dots (1)$$

in which Q = the discharge at Section $m = q x$; $q \doteq$ the constant increment of inflow per unit length of flume; w = the unit weight of water; g = the

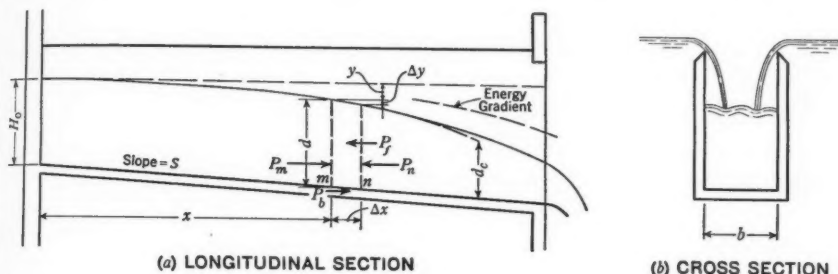


FIG. 1

acceleration of gravity; and V = the horizontal component of the mean velocity at Section m . The momentum at Section n for the mass per second is

$$M_n = \frac{(Q + q \Delta x) w (V + \Delta V)}{g} \dots \dots \dots (2)$$

The change in momentum in the distance Δx is

$$\Delta M = \frac{w}{g} [Q \Delta V + q \Delta x (V + \Delta V)] \dots \dots \dots (3)$$

or, neglecting the product $\Delta x \Delta V$ and expressing Equation (3) in terms of differentials,

$$dM = \frac{w}{g} [Q dV + q V dx] \dots \dots \dots (4)$$

Since $\frac{dM}{dt} = \frac{dM}{dx} \frac{dx}{dt} = V \frac{dM}{dx}$,

$$\frac{dM}{dt} = \frac{w}{g} \left[V Q \frac{dV}{dx} + q V^2 \right] \dots \dots \dots (5)$$

in which dM is the change in momentum in the time dt .

Equation (5) gives the time rate of change of linear momentum at any section, m , for the mass of water flowing per second. This is equal to the net force in the direction of x producing the change. The net force in the direction of x acting upon the volume of water between Sections m and n is the difference between the static pressures less the friction force. The static pressure at Section m is

$$P_m = \frac{b w d^2}{2} \dots \dots \dots (6)$$

and at Section n is

$$P_n = \frac{b w}{2} (d - \Delta y + S \Delta x)^2 = \frac{b w}{2} (d^2 - 2 d \Delta y + 2 d S \Delta x) \dots (7)$$

in which S = the slope of the bottom. The horizontal component of the static pressure on the bottom is

$$P_b = b w \left(\frac{2 d - \Delta y + S \Delta x}{2} \right) S \Delta x = w b d S \Delta x \dots (8)$$

The net static pressure on the water prism mn is the summation of Equations (6), (7), and (8), or

$$P_m - P_n + P_b = w b d \Delta y \dots (9)$$

It will be noted that the slope of the bottom has no effect upon the value of the net pressure. It may be shown also that a changing width b has no influence on the value of this force.

The friction drag is

$$P_F = F (b + 2 d) \Delta x = \frac{F b d \Delta x}{R} \dots (10)$$

in which F = the frictional drag per unit area of channel wall, and R = the hydraulic radius at Section m . The equivalent water head on the area bd corresponding to the friction drag P_F is:

$$\text{Head} = \frac{P_F}{w b d} = \frac{F b d}{w b d R} \Delta x = \frac{F}{w R} \Delta x \dots (11a)$$

and from the Weisbach-Darcy formula this lost head is also given by

$$\text{Lost head} = \frac{f \Delta x V^2}{4 R 2 g} \dots (11b)$$

Hence, from Equations (11),

$$F = w \frac{f}{8 g} V^2 \dots (12)$$

in which f = the Weisbach-Darcy friction factor. From Equations (10) and (12),

$$P_F = \frac{w f V^2 b d}{8 g R} \Delta x \dots (13)$$

The net accelerating force on the water prism mn , from Equations (9) and (13),

$$\alpha_{mn} = w b d \Delta y - w \frac{f V^2 b d}{8 g R} \Delta x \dots (14)$$

and since the volume of water between Sections m and n is

$$b d \Delta x = \frac{\Delta x}{V} Q \dots (15)$$

the accelerating force on the water flowing per second (that is, Q) is

$$\frac{V}{\Delta x} \left(w b d \Delta y - w \frac{f V^2 b d}{8 g R} \Delta x \right) = w Q \frac{\Delta y}{\Delta x} - w Q \frac{f V^2}{8 g R} \dots \dots (16)$$

Expressing Equation (16) in terms of differentials and equating to Equation (5):

$$\frac{dy}{dx} - \frac{f V^2}{8 g R} = \frac{1}{g} \left[V \frac{dV}{dx} + q \frac{V^2}{Q} \right] \dots \dots \dots (17)$$

Equation (17) is general for any shape of channel, and it is identical with the Hinds equation except for the friction term.

APPLICATION TO CONSTANT WIDTH FLUMES

In order to integrate Equation (17) it is convenient to express the velocity, discharge and draw-down, y , in terms of the dimensions of the channel. For

this purpose, $V = \frac{q x}{b d}$; $\frac{dV}{dx} = \frac{q}{b} \left[\frac{1}{d} - \frac{x}{d^2} \frac{dd}{dx} \right]$; and (since $y = H_0 + S x - d$),

$\frac{dy}{dx} = S - \frac{dd}{dx}$. Substituting these values in Equation (17) and simplifying, the equation becomes

$$\frac{dd}{dx} = \frac{2 q^2 x d - S g b^2 d^3 + \frac{f}{8 R} q^2 x^2 d}{q^2 x^2 - g b^2 d^3} \dots \dots \dots (18)$$

Equation (18) may be written as follows:

$$\frac{2 x d dx - x^2 dd}{d^2} + \frac{g b^2}{q^2} d dd = \frac{S g b^2}{q^2} d dx - \frac{f x^2}{8 R d} dx \dots \dots \dots (19)$$

The terms on the left side of Equation (19) integrate directly, and the partially integrated equation is as follows:

$$\frac{x^2}{d} + \frac{g b^2 d^2}{2 q^2} = \frac{S g b^2}{q^2} \int d dx - \frac{f}{8} \int \frac{x^2}{R d} dx + C \dots \dots \dots (20)$$

When $x = 0$, $d = H_0$, from which

$$C = \frac{g b^2 H_0^2}{2 q^2} \dots \dots \dots (21)$$

In the term including S in Equation (20) the expression under the integral is equal to the area in the distance x between the water-surface curve and the bottom. This area equals $\bar{d}x$, in which \bar{d} = the average depth throughout the distance x . An approximate integration of the friction term may also be had by assuming both R and d constant and equal, respectively, to \bar{R} and \bar{d} , where \bar{R} = the average hydraulic radius throughout the distance x . With these approximations, the completed integration is

$$\frac{x^2}{d} - \frac{g b^2}{2 q^2} (H_0^2 - d^2) = \frac{S x g b^2 \bar{d}}{q^2} - \frac{f x^3}{24 \bar{R} \bar{d}} \dots \dots \dots (22)$$

Solving for H_0 , Equation (22) becomes

$$H_0 = \sqrt{d^2 + \frac{2 Q^2}{g b^2 d} - 2 S x \bar{d} + \frac{f x Q^2}{12 g b^2 \bar{R} \bar{d}}} \dots (23)$$

Equation (23) may be used to solve for the value of H_0 if the value of d at any point in the channel is known or fixed. The solution requires, however, that the average values of R and d throughout the length considered be assumed for substitution in the equation. If the invert is level and the friction loss is small, the last two terms may be omitted. For an accurate solution of the value of H_0 , after the first approximation is made, the equation must be solved for d at several points along the channel in order to check the assumed values of \bar{R} and \bar{d} . Adjustments are then made by trial and error.

Equation (22) is a cubical equation in terms of d , and its solution for d is simplified if the equation is rearranged in dimensionless form as follows:

$$\left(\frac{d}{H_0}\right)^3 + \left(\frac{f Q^2 x}{12 g b^2 \bar{R} \bar{d} H_0^2} - \frac{2 S x \bar{d}}{H_0^2} - 1\right) \frac{d}{H_0} + \frac{2 Q^2}{g b^2 H_0^3} = 0 \dots (24)$$

or,

$$\left(\frac{d}{H_0}\right)^3 + A \left(\frac{d}{H_0}\right) + B = 0 \dots (25)$$

in which

$$B = \frac{2 Q^2}{g b^2 H_0^3} \dots (26)$$

and

$$A = \frac{f H_0 x B}{24 \bar{R} \bar{d}} - \frac{2 S x \bar{d}}{H_0^2} - 1 \dots (27)$$

The solution of Equation (25) for $\frac{d}{H_0}$ may be obtained graphically from Fig. 2 for various values of A and B .

CRITICAL DEPTH

The flow characterized by the foregoing equations is at the upper alternate stage. For most of the lateral spillway channels used in water and sewage plants, the depth at the lower end of the channel will be fixed at upper stage by the hydraulic conditions in the conduit down stream, or the flume will discharge freely. In the latter case, illustrated in Fig. 1, the pressure distribution near the end of the flume will be non-hydrostatic due both to aeration under the nappe and to the existence of appreciable vertical acceleration in this region. The region will extend for a distance three or four times the depth up stream from the lower end. For the design of freely discharging flumes, a close approximation to actual conditions will be obtained if a hydrostatic control section is assumed to exist at a distance up stream from the end equal to three or four times the critical depth. Since the value of the critical depth for a rectangular flume is

$$d_c = \sqrt[3]{\frac{Q^2}{g b^2}} \dots (28)$$

Equation (23) may be written in terms of the critical depth as follows:

$$H_0 = \sqrt{3 d_c^2 - 2 S x_c \bar{d} + \frac{f x_c d_c^3}{12 \bar{R} \bar{d}}} \dots \dots \dots (29)$$

For this case, the coefficients in the cubical formula, Equation (25), have values as follows:

$$B_c = 2 \left(\frac{d_c}{H_0} \right)^3 \dots \dots \dots (30)$$

and

$$A_c = - \sqrt[3]{\frac{27}{4} B_c^2} = - 3 \left(\frac{d_c}{H_0} \right)^2 \dots \dots \dots (31)$$

The value of B_c in Equation (30) was obtained by combining Equations (26)

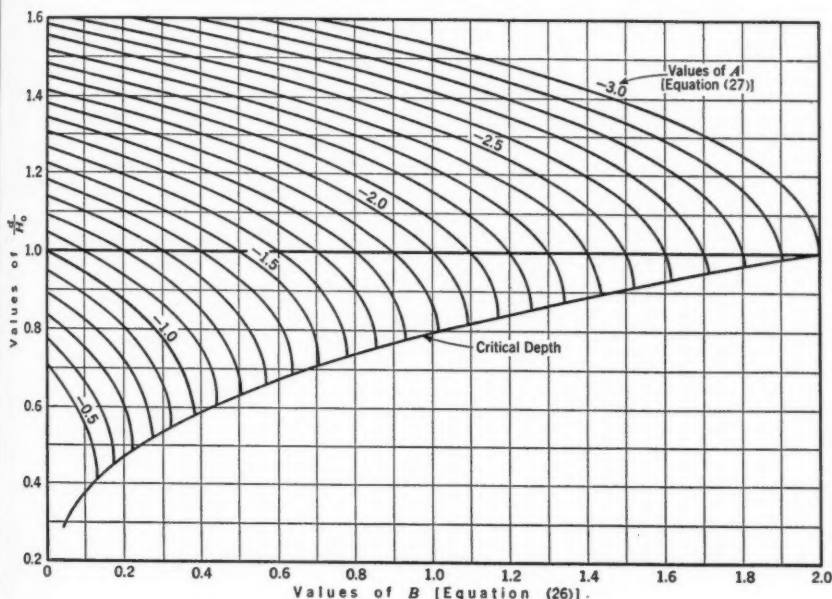


FIG. 2.—GRAPH FOR THE SOLUTION OF EQUATION 25

and (28) to eliminate $\frac{Q^2}{g b^2}$. The value of A_c in Equation (31) was obtained from Equation (25) by setting the derivative of B with respect to $\frac{d}{H_0}$ equal to zero and solving for A . Values of $\frac{d_c}{H_0}$ for various values of B_c or A_c are shown on the "critical depth" line of Fig. 2.

CROSS-SECTIONS OTHER THAN RECTANGULAR

The foregoing equations were developed for rectangular cross-sections, but they are also applicable to channels with other bottom shapes if the depth at

any section is taken as the average depth over the flume width. This may be demonstrated by consideration of Fig. 3 which shows a rectangular, a V-shaped and a semi-circular bottom. In all three cases, since the cross-sectional area is $b d$, the static pressure on the cross-section is

$$P = w b h_0 d \dots \dots \dots (32)$$

in which h_0 = the depth to the center of gravity of the cross-section. The value of h_0 , however, is not the same for the three cases.

For a rectangular cross-section, $h_0 = \frac{d}{2}$. For the V-shaped bottom,

$$h_0 = \frac{d}{2} + \frac{z^2}{6d} \dots \dots \dots (33)$$

and,

$$P = \frac{w b d^2}{2} + \frac{w b z^2}{6} \dots \dots \dots (34)$$

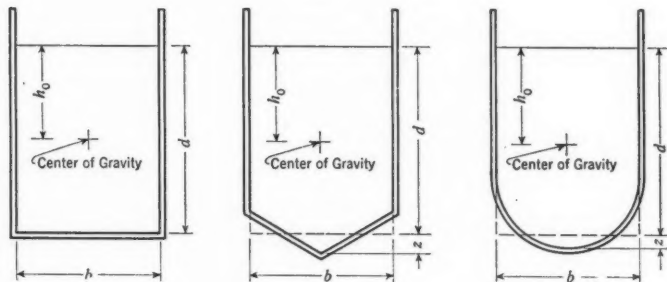


FIG. 3

The values of P_m and P_n are the same as those given by Equations (6) and (7) except that each contains the additional term $\frac{w b z^2}{6}$. The difference in pressures, therefore, is the same as for the rectangular flume and is given by Equation (9). For the semi-circular bottom,

$$z = \frac{4 - \pi}{8} b = 0.1073 b \dots \dots \dots (35)$$

$$h_0 = \frac{d}{2} + \left(\frac{1}{12} - \frac{\pi^2}{128} \right) \frac{b^2}{d} = \frac{d}{2} + 0.0062 \frac{b^2}{d} \dots \dots \dots (36)$$

and

$$P = \frac{w b d^2}{2} + 0.0062 w b^3 \dots \dots \dots (37)$$

In this case, as in the case of the V-shaped bottom, the second term in the equation for P cancels when P_n is subtracted from P_m ; and the net static pressure is given by Equation (9).

The critical depth is the same for any shape of bottom and is given by Equation (28), in which d_c is the average depth at the cross-section.

EXPERIMENTAL EVALUATION OF FRICTION FACTOR

Experimental measurements which are sufficiently complete and reliable for the evaluation of the friction factor are difficult to obtain. Since the theory is premised upon the assumption of a constant increment of inflow, it is imperative that the weir or weirs at the top of the flume walls be level. Moreover, because of the turbulence in the flume, an accurate measurement of the position of the water surface at any section is not possible. In order to counteract the effect of the errors in the measurement of the position of the water surface, the draw-down should be relatively great, and depth measurements should be made at three or more stations along the flume. The effects on the pressure distribution, of entrained air and of the momentum of the incoming water (both of which are of considerable importance for dam spillways), are negligible for the smaller channels used in water and sewage plants. The momentum of inflow results in increased friction factors due to increased turbulence.

The best set of experimental measurements available to the writer was made upon one of the wash-water gutters at the Springwells Water Purification Plant, City of Detroit, Mich., in 1931.⁴ This gutter is 2 ft wide with weirs 40.25 ft long. It is of cast iron with a semi-circular bottom and an invert slope of 0.25 in. to the foot. Measurements of the water surface and of the weir crest on both sides were made at intervals of 5 ft along the length of the flume. The maximum difference in elevation of the weirs at any two points was about 0.01 ft, which corresponds with a maximum variation in q of about 15% at the maximum discharge used.

Three runs were made at discharge rates of 10.91, 9.05, and 7.14 cu ft per sec, respectively. The water surface for the two larger discharge rates was above the semi-circular bottom, so that b was constant along the flume. The friction factor was evaluated by means of Equation (23) at 0.055 for both of these runs. Fig. 4, with Table 1, shows the relative position of the measured water surface and of the profile computed by means of Equation (25) and Fig. 2.

Fig. 5, with Table 1, shows the measured and computed water surface curves for the central concrete gullets of one of the filters at each of the two water purification plants at Cleveland, Ohio. The measurements at the Division Avenue Plant were made in 1920, and at the Baldwin Filtration Plant in 1928, both sets being under the direction of G. W. Hamlin, M. Am. Soc. C. E. The values of f (0.10 and 0.11) obtained by these measurements on two different gutters of similar size tend to substantiate the reliability of both sets of measurements. The inflow to main gutters is at intervals along the length and not continuous, as called for by the theory. Notwithstanding this fact, these measurements indicate that the theory may be used with success for gullets in which the points of inflow are not too widely separated.

Measurements of the water surface at two points were also made in lateral gutters for the same runs. These gutters have weir lengths of about 15 ft at both plants. The computations for f in the lateral gutter at Division Avenue resulted in a negative value, doubtless due to the fact that the weirs were about

⁴ Transactions, Am. Soc. C. E., Vol. 101 (1936), Fig. 9, p. 362.

0.03 ft lower at the down-stream than at the upper end. The computed value of f for the lateral gutter at the Baldwin Plant was approximately 0.02 for a discharge of 4.42 cu ft per sec. This value is probably unreliable because it is

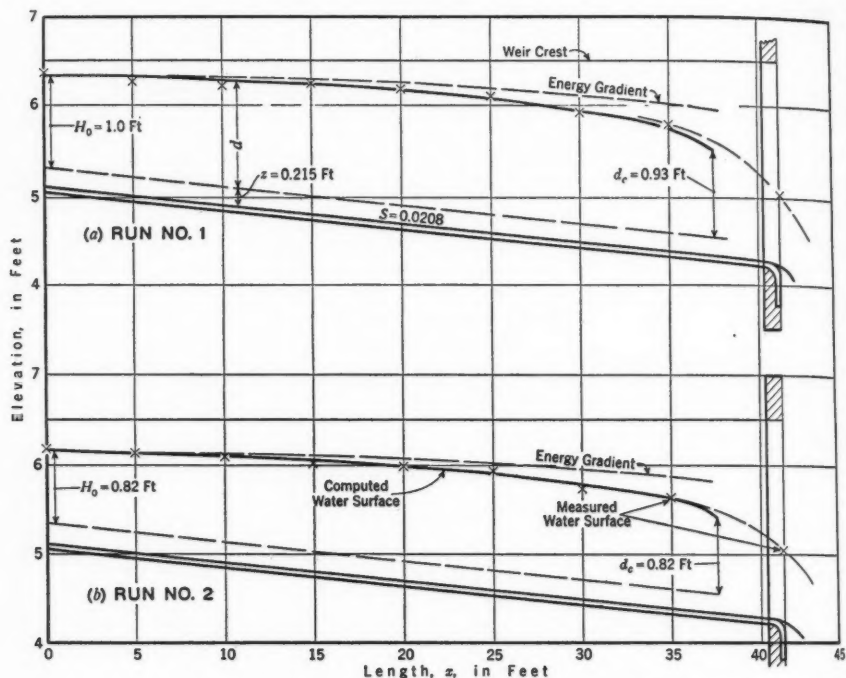


FIG. 4.—DETROIT (MICH.) EXPERIMENTS (SEE TABLE 1)

TABLE 1.—OPERATING DATA, TESTS ON LATERAL SPILLWAY CHANNELS

Figure No.	Test	Discharge, Q , in cubic feet per second	Increment of inflow, q , in cubic feet per second	Friction factor, f	Width, b , in feet
4(a)	Detroit, Michigan, Experiments:				
4(b)	Run No. 1	10.91	0.271	0.055	2.0*
	Run No. 2	9.05	0.225	0.055	2.0*
5(a)	Cleveland, Ohio, Experiments:				
5(b)	Division Avenue Plant	55.6	1.135	0.10	2.5
5(b)	Baldwin Filtration Plant	73.6	1.15	0.11	2.5
6	Massachusetts Institute of Technology:				
	Experimental Lateral Spillway Channel	1.237	0.0618	0.033	0.75

* Semi-circular bottom.

based upon only two measured depths along the channel and because the condition of the weirs was not observed.

A number of experiments were run on a lateral spillway channel at the Massachusetts Institute of Technology under the direction of the writer.

This channel is of wood with rectangular cross-section, 9 in. wide, and with a level bottom. The inflow is from one side over a weir 20 ft long. Fig. 6 illustrates the results of one of the most reliable runs on this channel, made by

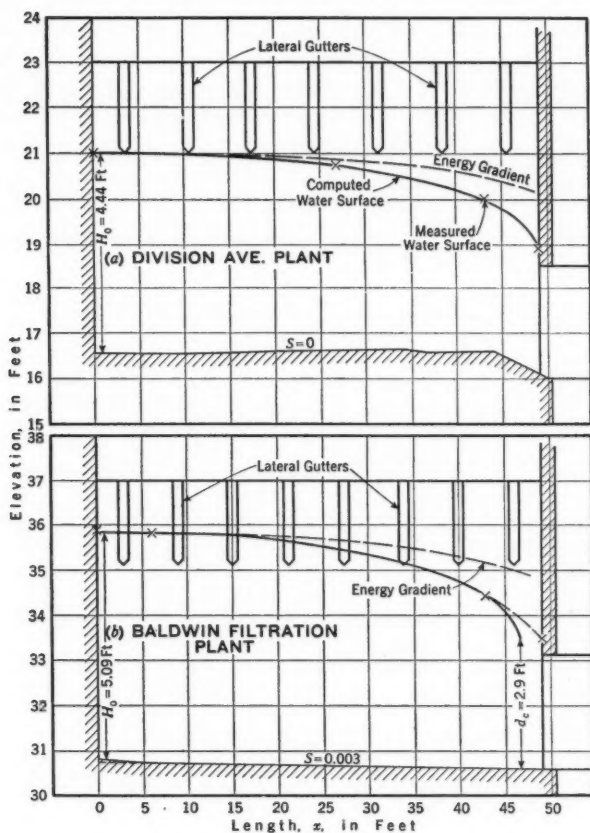


FIG. 5.—CLEVELAND (OHIO) EXPERIMENTS (SEE TABLE 1)

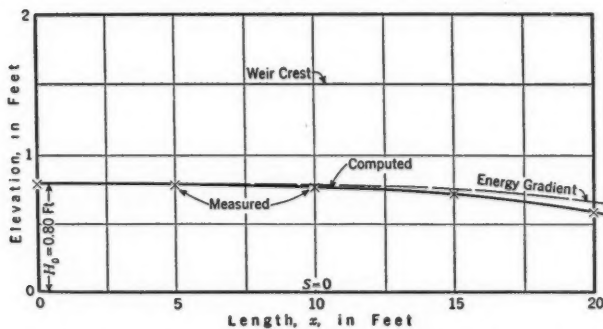


FIG. 6.—EXPERIMENTAL LATERAL SPILLWAY CHANNEL (SEE TABLE 1)

Mr. E. A. Kass.⁵ The variation in level of the weir was only about 0.001 ft. The head on the weir was approximately 0.07 ft and the nappe fell freely. The computed value of f for this run was 0.033.

ACKNOWLEDGMENT

The writer is indebted to Mr. Hinds for the development of the basic differential equation which has been used in this paper. The experimental data from the Springwells Filter Plant in Detroit were furnished through the courtesy of Arthur B. Morrill, M. Am. Soc. C. E. The Cleveland data were furnished through the courtesy of J. W. Ellms, M. Am. Soc. C. E., and Mr. Hamlin. The writer also wishes to acknowledge the valuable assistance of a number of his students who made experimental investigations of flow and pressure characteristics.

⁵ "Loss of Head in Lateral Spillway Channels," by E. A. Kass; presented to Massachusetts Institute of Technology in 1937, in partial fulfillment of the requirements for the degree of Bachelor of Science.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

MALARIA CONTROL FOR ENGINEERS

REPORT OF THE JOINT COMMITTEE
OF THE NATIONAL MALARIA COMMITTEE
THROUGH ITS SUB-COMMITTEE ON ENGINEERING
AND

THE SANITARY ENGINEERING DIVISION
AMERICAN SOCIETY OF CIVIL ENGINEERS

INTRODUCTION

Malaria is a problem peculiar to those parts of a country where climatic conditions favor the development of the malaria parasite and the *Anopheles* mosquito.

A large part of the malaria in many communities is man-made and preventable, at a price. Man-made malaria is a by-product of construction works. The engineer, as designer and supervisor of such works, is therefore often directly responsible for an increase in malaria prevalence, or for its continued endemic form, when a fundamental knowledge of the cause of the disease and the application of principles of control could prevent such artificial stimulation or continuation.

With a desire to provide engineers with this information, the National Malaria Committee, in 1930, authorized the appointment of a Sub-Committee on Engineering to prepare a series of papers to serve as lecture courses in engineering colleges. After the resulting work had progressed for some time, the Sanitary Engineering Division of the Society became interested and expressed willingness to be identified with the project. Thus the present report appears under the combined sponsorship of both organizations, as represented on the joint committee.

The report contains fundamental information that should be a part of the curriculum in every course leading to a degree in any of the branches of civil engineering. More specific data on procedure in malaria control drainage are often desirable, and this is covered very briefly in the latter part of this report. For those requiring further reference, a bibliography is presented in Appendix I. Most of the references are found in pamphlets and periodicals,

NOTE.—Written discussion of this report will be transmitted directly to the chairman for the information of the Committee.

some of which are now unfortunately out of print, but all may be consulted at most of the larger libraries.

Malaria is a term applied to a group of diseases caused by certain organisms known as the malaria parasites. These diseases are characterized by regular recurrence of fever at definite intervals of 24, 48, or 72 hr. It is spread only by one particular group of mosquitoes known as "anophelines." As there are a number of species of anophelines that differ somewhat in habits, the methods of their control in different countries will vary with the species that are capable of conveying malaria in the particular locality under consideration.

There is a difference in range of flight, in selection of breeding places, in tendency to go into occupied residences, etc., with different species of anophelines. Fortunately, in the United States malaria is conveyed chiefly by one species, known as *Anopheles quadrimaculatus*. West of the Rocky Mountains there is another species, *Anopheles maculipennis*, that also transmits malaria. Although other types are present in this country, they are not known to be of public health, and economic, importance.

There is no question but that in regions of relatively high malaria prevalence this disease holds back the general development of the country affected and interferes with the growth of local industries. Malaria is a menace to the people of any country in which it has a decided incidence. In the amount of sickness and suffering, the loss of time and efficiency, the expense, the lowered vitality of those afflicted, and in the reduction of valuation of property, malaria is without a rival among the diseases affecting Man.

The death rate from malaria in temperate climates, probably averaging 0.5% of the cases, may not appear alarming, yet the loss of efficiency is often of most decided importance. In malarious sections of the United States the loss of efficiency caused by malaria is greater, beyond comparison, than that caused by any other disease. The man infected with malaria is frequently half sick all the time.

The control of malaria often becomes a labor problem of the first importance in proportion to its proximity to the tropics, and for centuries it has continued to hinder the progress of semi-tropical and tropical countries.

Agricultural development is conceded to be of prime importance, and in the United States the highest malaria incidence occurs among the farming population. In a study of malaria prevalence on a well-managed plantation in Louisiana, the United States Department of Agriculture found, from May to October, 1913, a total loss of time of 1 066 days in 74 tenant families, due to malaria, 60 families would have done the same work had there been no malaria. The actual loss of time amounted to \$2 200, and the loss of efficiency to \$4 300.

In 1926, surveys were made in two counties of another State to get an idea of economic losses caused by malaria. In the first county the annual losses due to malaria prevalence amounted to approximately \$3 per acre under cultivation; and in the second county the annual losses per acre caused by malaria amounted to about one-third of the annual assessed county tax.

Similar financial losses have been found in industrial plants operating in localities of high malaria incidence. The managers of the cotton mills at

Roanoke Rapids, N. C., reported a loss of efficiency from 40% to 60% among their employees during the four malaria months, and as a result of the high malaria incidence, a large part of the cotton mill population were transients. A large lumber mill in east Texas was able to increase its output 20% after control of its malaria problem (50).¹

As one travels south from the southern border of the United States a more severe type of malaria is found, and the problem of its control becomes more important. In some of the larger American industrial corporations operating in the American tropics, one-half of the hospital cases are caused by malaria, and companies with physicians and engineers who have been sufficiently alert to give malaria control the attention it merits have profited accordingly. In a report of the United Fruit Company the camp manager reports: "By means of malaria control and camp sanitation we have obtained more working days per man, more stability, less sickness in camps, and in a three-year period the average increase of earning per man per month has been from \$30.79 to \$40.80." Such increase is practically 33% increase of labor output and is well worth consideration in engineering construction operations. Unfortunately, the directing engineers of some corporations have not yet "seen the light," and as a result some American-owned and American-directed corporations are found operating in less difficult semi-tropical territory with from 30% to 80% of their working forces infected with malaria. This labor efficiency loss is well worth serious attention. Such conditions were observed in recent years in the Mexican oil field area. No attempt was made to prevent the spread of malaria from the sick to those who were well. Some of the camp sites could not have been better chosen had it been intended to see how rapidly the efficiency of the workmen could be decreased, and even where large labor camps could have been made free of the malaria-conveying mosquito by a relatively low expenditure for drainage, it was apparent that the engineers in charge did not understand the value such work would have returned in labor reduction costs.

Experience has taught employers of labor to take into consideration as causes for rejection in employment such conditions as alcoholism, epilepsy, diabetes, etc., particularly in filling positions where danger and responsibility are involved. Conditions responsible for mental aberrations or loss of consciousness may be a contributing factor in serious loss of life and property. Malaria may become, and has been important as, a potential cause for accidents.

In a number of States malaria has seriously affected the cost of engineering construction operations, due to delayed construction, insufficient labor, labor shortage or replacement, and seasonal labor turnover; and in tropical countries it has rendered costs prohibitive, or has made it necessary to carry additional labor and engineering supervision.

In engineering operations in the United States, such as impounded water projects, irrigation projects, railroad and highway construction, it is not unusual for labor to be recruited from malarious sections and taken to construction camps, causing epidemic outbreaks in the camp and vicinity. In connection

¹ For reference to numerals in parentheses, see Appendix I.

with large hydro-electric projects this has resulted in serious lawsuits brought by farm owners in territory adjacent to the construction projects. There are numerous records where this disease has been a source of economic loss in connection with canal construction, railroad maintenance, hydro-electric development, sawmill and logging camp operations, as well as other commercial industries (50).

The engineer can, and should, consider the potential dangers of malaria in connection with survey parties. In selecting the personnel and labor, it is often advisable to know whether or not malaria-infected men are sent to a new camp where they will become a menace and the cause of a number of the party becoming infected. In the boundary survey parties on the Panama Canal this factor became one of prime importance.

When camp sites are being selected their relation to nearby sources of *Anopheles* should be considered, and also their proximity to native houses or villages and the malaria status of the latter in relation to becoming a means of infecting those who live at the camp. Where malaria prevails it is a good rule to have the engineers' camp a mile from native villages, so that *Anopheles* infected at the village will not reach the camp.

The old idea that a camp on a hill is safe has been discarded; and in potential malaria districts the flight range of malaria-conveying mosquitoes, as well as the potential sources of those mosquitoes near the camp, and the cost and possibilities of drainage, should be considered.

In uninhabited territory, when a camp must be located close to swampy areas where profuse breeding of *Anopheles* occurs, it is decidedly important to be sure no malaria carriers are found among the force in camp. The effective screening of the camps is an additional precaution that is well worth consideration. In temporary labor camps at Panama, where no other precautionary measure was practicable, it was thought advisable to employ a boy to destroy blood-gorged *Anopheles* each morning, and it was found that a camp so protected had only one-fortieth of the malaria sick rate of a nearby camp not so protected.

During the World War precautionary anti-malaria measures were taken at the military camp areas. Some of these camps were in the same locations as previous military camps of the Spanish-American War period. The precautionary measures taken so reduced the malaria sick rate that it was only 0.5% of that of the previous war-period record. This protection of field forces was accomplished by engineers and medical officers interested in malaria reduction, and indicates what can be accomplished.

In planning drainage to reduce the prevalence of malaria mosquitoes it must be remembered that the drainage ditches themselves are potential sources of mosquitoes—that it is a mistake to have ditches with flat, wide bottoms where they can be avoided. The bottom cross-section of the ditch should be narrow, thus insuring sufficient velocity of residual water to prevent the mosquito from laying her eggs and to remove any mosquito larvae washed into the ditch during showers.

In lining ditches with concrete in suburbs of cities, a common mistake is to have a flat cross-sectional bottom instead of a bottom central channel, or rather

a ditch within a larger ditch. Wide, flat-bottom, concrete-lined ditches often become a prolific source of the malaria mosquito. In such ditches sufficient refuse will collect to make the water practically stagnant in many parts of the ditch.

ANOPHELINE MOSQUITOES AND MALARIA TRANSMISSION

Malaria parasites are minute Protozoa which require two hosts, Man and the anopheline mosquito. There are three species of the malaria parasite, *Plasmodium vivax*, *P. malariae*, and *P. falciparum*, each of which is the causative agent of a specific type of malaria, known as tertian, quartan, and aestivo-autumnal (sub-tertian, or malignant) respectively. All are transmitted in the same manner.

In Man the parasites live in the red blood cells where they multiply asexually; that is, reproduction is accomplished without fertilization, and the development in Man is termed the asexual cycle (see Fig. 1). The young organisms are termed *trophozoites*; the older ones, *schizonts*. When mature the *schizont* divides into a number (from 8 to 24) of small cells or segments termed *merozoites*. Upon rupture of the corpuscle these are discharged into the blood stream and attack new cells in which the cycle is repeated. The cycle of asexual development for each of the three species is: *P. vivax* (tertian), 48 hr; *P. falciparum* (aestivo-autumnal), 24 to 48 hr; *P. malariae* (quartan), 72 hr.

While this multiplication proceeds there appears another stage in the cycle of the parasite. Some of the young organisms develop into male cells (microgametocytes) and others into female cells (macrogametocytes). When an anopheline mosquito bites a person harboring these sexual forms, some of them are carried into the mosquito's stomach with the blood which it sucks up. In the stomach of the mosquito the parasites become sexually mature and are then termed *gametes*, the male cell being called a *microgamete* and the female, a *macrogamete*. These conjugate to form an *ookinete* which, after certain changes, penetrates the stomach wall of the mosquito. The ookinete here comes to rest, becomes incased in a thin cyst wall, and is then termed an *oocyst*. The oocyst absorbs nutriment from the surrounding cells and undergoes a remarkable development by which it becomes filled with an immense number of minute motile organisms called *sporozoites*. By bursting of the oocyst the sporozoites are liberated into the body cavity of the mosquito and are carried by the circulation to the various parts of the mosquito's body. Large numbers of them reach the salivary glands and are injected into Man by the mosquito's bite. The sporozoites penetrate the red blood cells and the life cycle begins anew.

The malaria parasite takes from 8 to 14 days or more, depending on temperature, to complete its development in the mosquito. After a person has been bitten by an infected mosquito the time usually elapsing before an attack of malaria occurs will vary as much as from 9 to 21 days or more, depending on the species of parasite concerned and other factors.

The foregoing indicates why the anopheline mosquito is essential to the transmission of malaria. It follows, therefore, that practical control measures

resulting in the reduction or elimination of these mosquitoes will go far toward controlling the spread of the disease.

Not all anopheline mosquitoes are important as malaria carriers, and some are apparently efficient carriers in one region but not in another. There is also

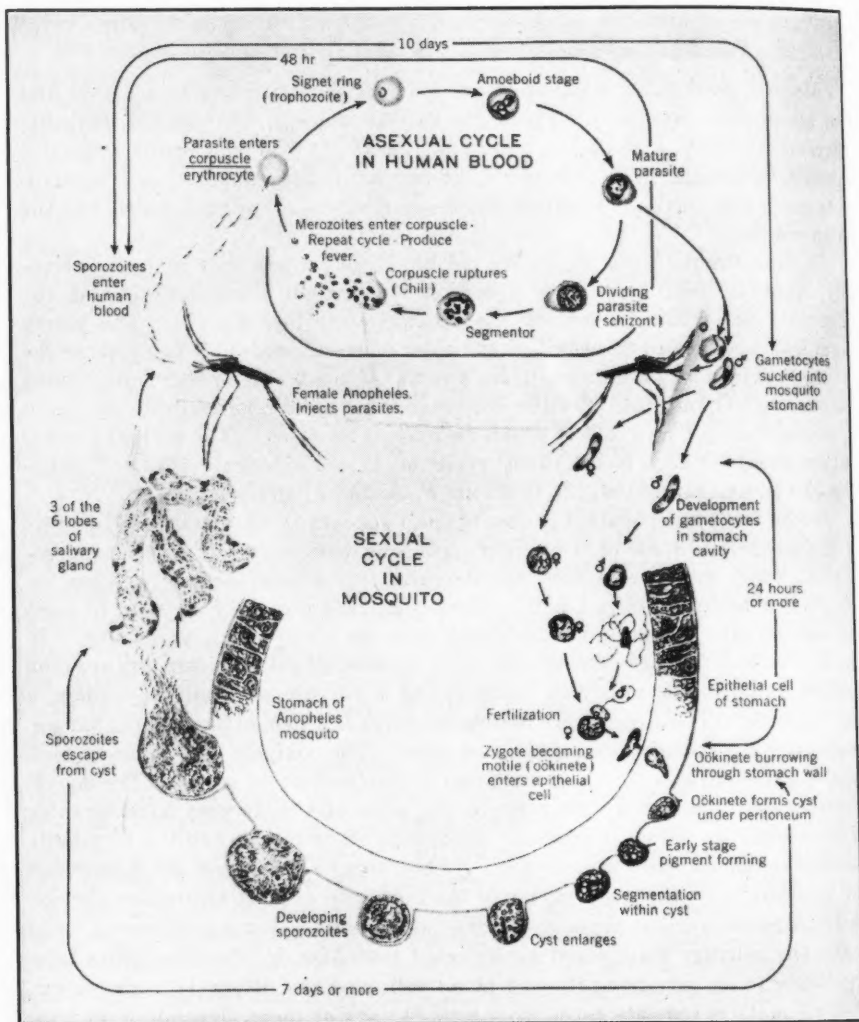


FIG. 1.—LIFE HISTORY OF THE MALARIA PARASITE, IN MAN AND IN THE *ANOPHELES* MOSQUITO

a great diversity in the individual habits of anophelines, particularly with regard to the places they select for breeding. Prior to instituting anti-mosquito measures for malaria control in any region, it is imperative that the exact habits of the species concerned be determined. It is desirable, of course, to

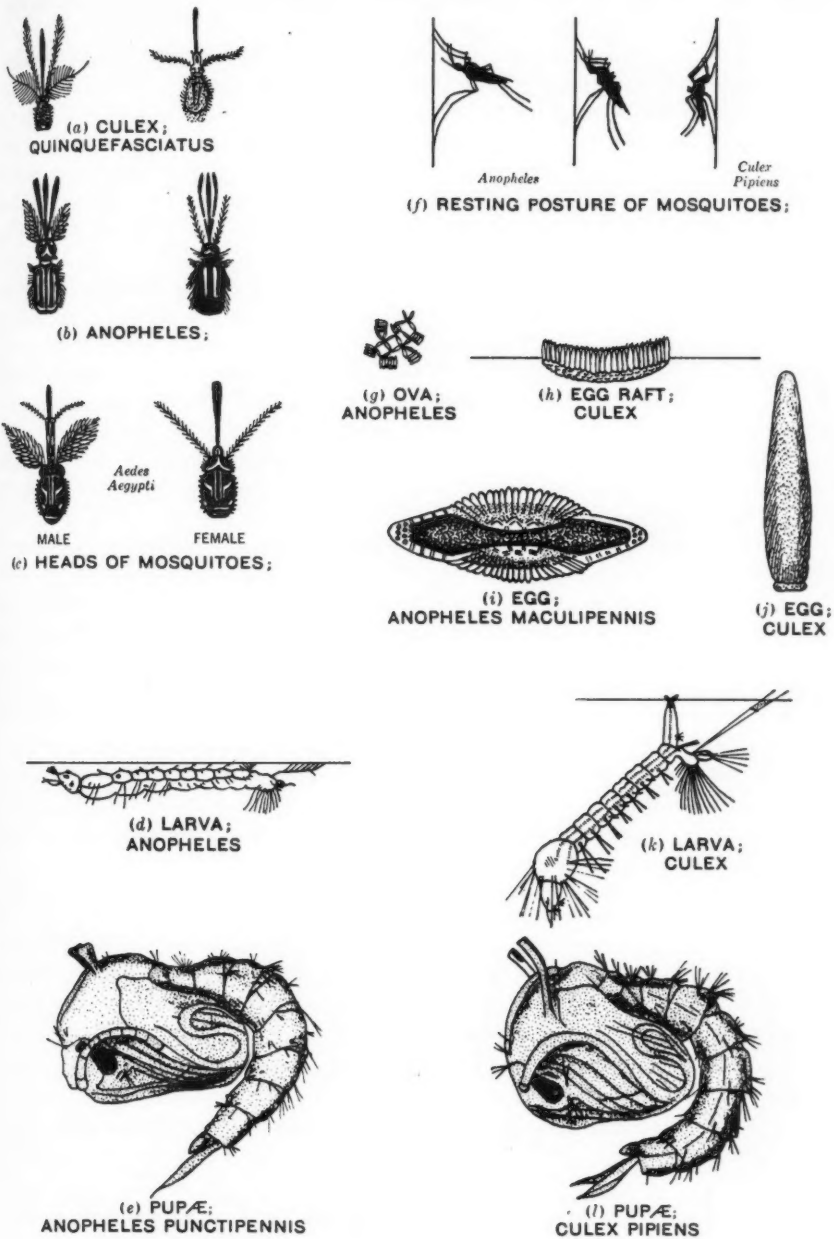


FIG. 2.—FIELD IDENTIFICATION OF MOSQUITOES

eliminate as many breeding places of other mosquitoes as possible during the course of malaria mosquito control.

Life History of Mosquitoes.—In common with other mosquitoes, anophelines have four distinct stages in their development, or life cycle. The eggs of anophelines are laid singly on the surface of water and hatch in 2 or 3 days into larvae or wigglers. The larval stage lasts for a week or more, depending on temperature and available food. When this stage is complete the larva transforms into a pupa. Both the larval and pupal stages are aquatic. The pupal stage lasts 2 or 3 days at summer temperatures, and from the pupa the adult emerges.

In general, successive generations of anophelines occur continuously during the warmer months of the year. The winter is usually passed by hibernating females in various types of shelters; however, there appears to be no true hibernation of anophelines in the Southern United States, as in warm periods during the winter the adults may become active and developing larvae may be found.

Only the female mosquito bites; the male mosquito obtains food from fruit and plant juices. In order to produce "hatchable" eggs it is usually necessary for the female to have had a blood meal.

Field Identification of Mosquitoes.—Adult *Anopheles* can be distinguished from other mosquitoes in several ways. As a rule the body of the anopheline mosquito is held at an angle approximating 45° from the resting plane (see Fig. 2(f)), whereas other mosquitoes hold the body more or less parallel to the surface on which they rest. Anopheline mosquitoes, with few exceptions, have definite spots in the wing pattern, whereas non-anophelines, as a rule, have wings with either concolorous scales or a mixture of contrasting scales without definite pattern.

Anopheles larvae lie parallel to the surface of the water (Fig. 2(d)); other mosquito larvae hang head down at an angle to the surface (Fig. 2(k)), from which they are suspended by a breathing siphon. *Anopheles* have no perceptible siphon and they rest and feed at the water surface.

To be able to distinguish anopheline mosquitoes beyond question, a certain degree of familiarity with insect morphology is essential. This knowledge can be obtained by a study of one of the many available bulletins on mosquitoes, or from general textbooks on entomology. M. F. Boyd has summarized the points by which the four stages of anopheline mosquitoes are distinguished from all other mosquitoes (10), as follows:

	Anophelini	Culicini (broad sense)
Imago	Palpi of both male and female long, in the former clubbed at tip, in the latter as long as the proboscis. Scutellum not lobed.	Palpi of female shorter than proboscis, of male long or short, in the former case slender at the tip. Scutellum posteriorly trilobed.
Ovum	More or less boat-shaped with lateral floats, laid singly.	Elongated, ellipsoidal or conical, without lateral floats, laid singly or in rafts.

Larva	Lying horizontally in the water just below the surface film, with which it is in contact at several points. Siphon absent.	Maintaining contact with the surface film only with the siphon, the body hanging obliquely or vertically downward. Siphon present, usually well developed and elongated.
Pupa	Siphons short and scoop-shaped, split down the front.	Siphons broadly conical or elongated tubular, unsplit.

Specific Identification of Anophelines.—The specific identification of *Anopheles* is extremely important because in malaria control it is necessary to direct operations only to those areas which are producing the species concerned with the transmission of the disease. This principle has been termed "species control of malaria." Reliable specific identifications of *Anopheles* may be secured by forwarding specimens to the Bureau of Entomology and Plant Quarantine of the U. S. Department of Agriculture at Washington, D. C. Because of variability of wing and leg markings, accurate identification of some female anophelines is often impossible. Male adults bred out from larvae and pupae are readily identified by the specialist. Many States also maintain departments of entomology where insect specialists will make the identifications or forward the specimens to Federal agencies.

Field Collecting.—The eggs of anophelines are small and are scattered over a breeding area. They are difficult to find, and search for them is a tedious process. The breeding places are usually located by finding the larvae.

Since mosquito larvae are aquatic organisms they must be searched for in collections of water. It is a common occurrence, even in this enlightened age, for a worker to be accosted, while collecting, by uninformed and apparently intelligent persons, with the inquiry as to what he is looking for. Upon replying that mosquito larvae are the object of search, anything from a soft titter to a loud guffaw may be heard, and the collector is informed that mosquitoes raise "only in grass, chinaberry trees, the tops of palmettoes, and the like." A demonstration of the presence of the larvae in the water may or may not convince such a person that he is wrong.

A satisfactory implement for use in collecting anopheline larvae is an ordinary white-enamelled water dipper having a diameter of about 5 in. The dipper handle should be tubular so that its length can be increased several feet by the insertion of a cane or stick. The dipper is used in two ways for collecting: (1) In areas having horizontal vegetation such as algae, lemna, and the like, a short sweep of the dipper across the surface of the water will skim up larvae if they are present; and (2) where there is considerable emergent vegetation the dipper is pushed gently against this vegetation with the lip just below the surface in such a manner as to allow the water to run in. It is important that the water be disturbed as little as possible where the dip is to be made, since the larvae and pupae, particularly the latter, are easily alarmed, and will drop quickly to the bottom of the water. A little practice in dipping will soon accustom one to the requisite motion and to the length of the sweep to be made without over-filling the dipper, thereby losing part of the catch, or including so

much trash as to make the finding of larvae in the dipper difficult. After being skimmed up in the dipper the larvae and pupae usually return quickly to the surface of the water. The pupae are more active than the larvae but are easily located by their peculiar bobbing movement when disturbed in the dipper. For transferring them from the dipper to the collecting bottle an ordinary medicine dropper may be used but should have the tip cut off so as to enlarge the opening. After cutting the tip of a medicine dropper the end should be filed or annealed so as to smooth the sharp edges. Ordinary 8-oz, wide-mouthed bottles, fitted with cork stoppers, make convenient containers for field use when it is desirable to bring the catch to the laboratory for rearing. If the species are sufficiently well known so that dead larvae only are needed for identification, the larvae may be transferred to small vials of preserving fluid. Ethyl alcohol, 70% to 80%, to which has been added a small amount of glycerine, is satisfactory for this purpose. As the hairs are used for classification it is important that few of them, if any, be broken off the specimens. If it is desired to send the samples to a specialist for determination, the vials may be wrapped in cotton and shipped in a mailing tube.

The specimens will reach their destination in a much better state of preservation if the preserving fluid completely fills the vial, expelling all air, and the stopper is tied on tightly with a bit of heavy thread and sealed with paraffin to prevent evaporation. For positive determination, large (4th stage) larvae are necessary and any collection sent for identification should include an ample number of specimens, together with a label showing the locality, date, and the collector's name.

Not all anopheline larvae can be reached by dipping. Hoof prints, for example, can best be examined by direct observation of the water surface. Muddying the water in these and other small collections of ground water is often an aid in detecting the larvae.

In rearing larvae to obtain adult mosquitoes or to get the later stages for identification, they should be placed in open shallow pans rather than left in bottles. As the larvae pupate the pupae should be transferred to vials or test-tubes containing a small quantity of water and a plug of cotton placed in the top so as to hold the adult when it emerges. If it is desired to rear a large number of larvae, particularly if the specimens are in the earlier stages, it will be necessary to supply ample food. Yeast, hay infusion, scum, and algae are the materials ordinarily used in feeding the larvae. The water in the rearing pans should be changed frequently to prevent fouling. Reared adults should be allowed to live at least 24 hr before killing; otherwise the body will shrink and this may prevent identification.

Adults.—Adult *Anopheles* are chiefly nocturnal in their habits. Their activity begins shortly before sundown and they remain active throughout the night. On cloudy days, however, or when confined in places where they cannot satisfy their blood lust at will, they will bite during the daytime. Of the common anophelines of eastern North America, *Anopheles punctipennis* (Fig. 3) is reported as biting more readily in the daytime than *A. quadrimaculatus*, and *A. crucians* is reported as biting in the daytime in sunshine as well as in shade.

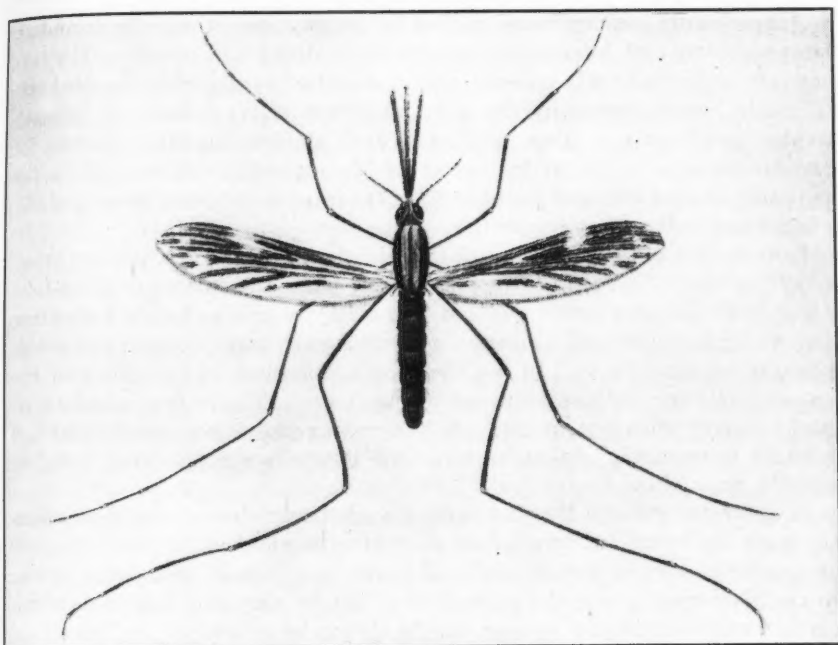


FIG. 3.—*ANOPHELES PUNCTIPENNIS*

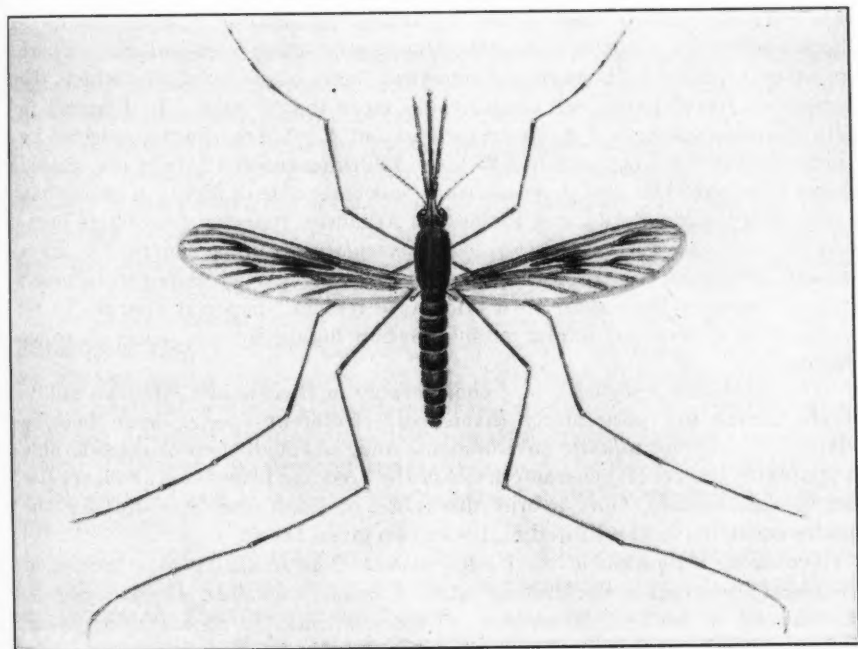


FIG. 4.—*ANOPHELES QUADRIMACULATUS*

Anopheles are usually found resting in the daytime in dark or semi-dark places which are cool, humid, and out of reach of strong air currents. The daytime resting places to which the attention of collectors should be directed are: (1) Inside houses, especially the sleeping rooms and porches; (2) beneath elevated dwellings and other buildings; (3) in stables and other shelters for domestic animals; (4) in privies; (5) under bridges and in culverts; (6) on the shady side of road-cuts and the like; (7) in the interior of hollow trees, and (8), in boxes and in barrels under shelter.

For catching anophelines a satisfactory implement is the "chloroform tube" or "killing bottle." This may be prepared by filling the bottom of a test-tube or long shell vial to a depth of about 1 in. with cut rubber bands, saturating these with chloroform and then covering with a small plug of cotton and a disk or two of blotting paper. In use the tube is uncorked and placed over the mosquito, allowing it to be overcome by the fumes. The tube should be kept tightly corked when not in use. As the resting places are usually dark, a flashlight is required. At night when the insects are active they may be collected while biting.

In order to preserve the distinguishing characteristics of the mosquitoes they must not be rubbed, crushed, or allowed to become moist. For shipment the specimens may be put in small pill boxes on a wisp of absorbent cotton. Do not place cotton over the mosquitoes. The locality and date of capture, together with the collector's name, should always be written on the box.

Flight Habits.—A knowledge of the distance of flight of the various species of *Anopheles* that convey malaria is of great importance in control operations. It is generally considered now that control of production within a radius of about 1 mile will ordinarily protect the human population from malaria. There are several instances, however, of observed flight of *Anopheles* in which the distances covered have been considerably more than 1 mile. In Panama in 1913 immense numbers of *A. tarsimaculatus* and *A. albimanus* were observed by LePrince to cover a distance of 6 250 ft. LePrince and Griffiths in the United States have observed that *A. quadrimaculatus* is capable of flights of more than a mile, and Geiger, Purdy and Tarbett, in Arkansas, reported a probable invasion of their control area by *A. quadrimaculatus* from a source 1.7 miles distant. Some of the tropical species have been reported as covering much greater distances than these. In temperate regions, dispersal appears to be much more widespread during prehibernation flights than is usual at other seasons.

The species of *Anopheles* vary considerably in their habits, and the habits of the larvae are particularly diversified. Different species have become adapted to different aquatic environments and, although there is considerable overlapping, the general characteristics of the breeding places of a given species can be determined. Only a brief discussion of these characteristics for the species occurring in the United States can be given herein.

***Anopheles* Mosquitoes of the United States.**—The following nine species of *Anopheles* are extant in the United States: *A. quadrimaculatus*, *A. punctipennis*, *A. crucians*, *A. walkeri*, *A. atropos*, *A. maculipennis*, *A. pseudopunctipennis*, *A. barberi*, and *A. albimanus*.

By experiments, most of these have proved to be susceptible to malaria infection, and several of them have been found infected with malaria parasites under natural conditions. From all the evidence available, however, it is generally agreed that only two of these species, *Anopheles quadrimaculatus* and *A. maculipennis* (see Fig. 4), are of importance as vectors of malaria in the United States. The others appear to be either too rare to play important rôles or their blood-feeding habits are such that they are not closely associated with Man. There is little evidence that *A. crucians*, *A. punctipennis* or *A. pseudopunctipennis* have been of much importance as vectors in the United States, although the last named species, at least, is believed to be an efficient carrier in Mexico and Argentina.

Breeding Habits and Distribution.—In the United States, therefore, with malaria control by anti-mosquito measures in view, the principal efforts should be directed against the two species, *A. quadrimaculatus* and *A. maculipennis*. The former occurs throughout the Southern and Southeastern States, and as far north as New York and Illinois; and *A. maculipennis* occurs from California to Alaska and eastward through Canada and the northern border of the United States to the Atlantic. It is also found in the Rio Grande Valley in New Mexico.

The important malaria transmitter in the South and Southeast, *Anopheles quadrimaculatus*, is usually termed a "pond breeder," as it is rarely found in flowing water. Fresh-water pools, ponds, shallow lakes, rice fields, borrow-pits, lake margins, ponded swamps, and places of like character, in which there is an abundance of vegetation and débris, are its preferred habitat. Apparently it prefers neutral or slightly alkaline water rather than acid water.

Anopheles maculipennis, the most important malaria carrier of the Far West, breeds by preference in shallow sunlit pools of clear water containing green algae. S. B. Freeborn states that for this species "hoof prints, wayside pools, neglected irrigation or drainage ditches, and seepage areas, furnish the most favorable locations" and that "this species is characterized in the West by a strong migratory flight in February in which the over-wintering females infest wide areas far beyond their normal breeding range. A less noticeable flight of the same individuals takes place in October preceding the period of semi-hibernation in which they pass the winter."

Anopheles crucians occurs principally in the coastal region of the United States from New York to Texas, and in the lower Mississippi Valley. Two races of this species are present in the United States, one inhabiting fresh water and the other salt water. The fresh-water race breeds (see Fig. 5) by preference in pooled intermittent streams, old borrow-pits, cypress swamps, and grassy ponds. It appears to select slightly acid waters. The salt-water race is found in salt marshes and is able to tolerate a fairly high percentage of salt in its breeding water.

Anopheles punctipennis occurs in southern Canada and throughout the United States. It breeds chiefly in pooled intermittent streams, seepage ponds, and the more permanent rain-water pools. This species appears to be the only one of the North American anophelines that is commonly found in streams.

Anopheles pseudopunctipennis has been found in California, Arizona, New Mexico, and western Texas, and has been reported once from Tennessee. It breeds in wayside sunlit pools, especially the semi-permanent ones left by seasonal rains in semi-arid country. In California it breeds in company with *Anopheles maculipennis* "and continues to breed for some time after these pools have become too foul for the latter" (7).

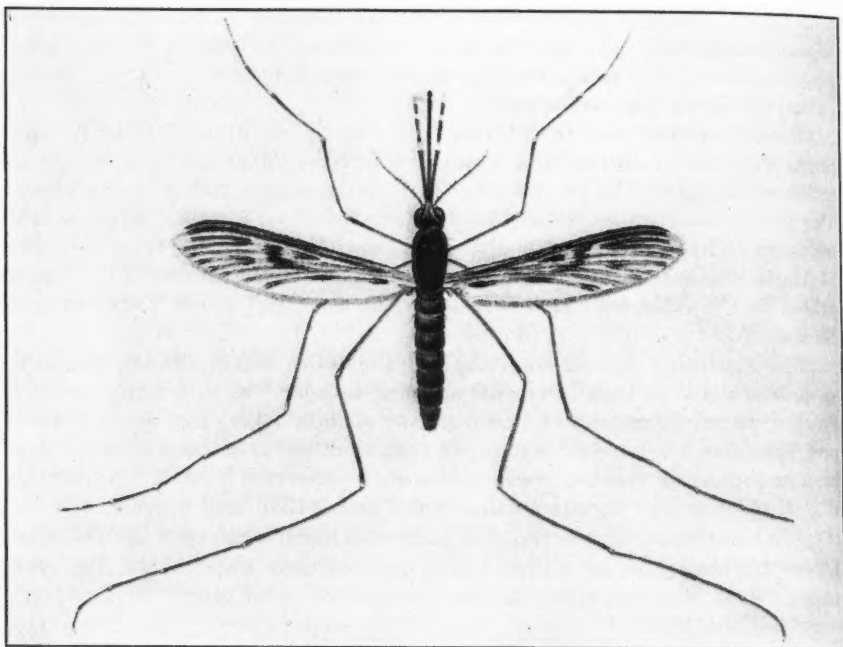


FIG. 5.—*ANOPHELES CRUCIANS*

Anopheles walkeri is rather rare and few records of its breeding places are available. Apparently it breeds in permanent or semi-permanent water containing much vegetation. It has been found in water covered by waterhyacinth (Mississippi), in cattail marshes (New York and Florida), and in rice fields (Arkansas).

Anopheles atropos is likewise a species seldom encountered. It breeds in brackish or salt-water pools along the Gulf and Atlantic Coasts of the United States, having been reported from the coastal areas of Louisiana, Mississippi, Alabama, Maryland, Virginia, and Florida. The larvae have been taken in grassy or algae-filled pools on the marshes and the shallow water on muck soils.

Anopheles barberi occurs throughout the southeastern part of the United States but is extremely rare. It is an example of an anopheline having a very specialized environment, as it breeds almost exclusively in water in tree holes.

Anopheles albimanus is of extremely rare occurrence in the United States although it is an important species in tropical America. It was found on one

occasion at Key West, Fla., and it has been reported from the lower Rio Grande Valley in Texas. Its breeding habits in the United States have not been observed.

Vegetation and Mosquito Breeding.—Much interest is aroused from time to time by reports that various plants have been found to inhibit anopheline breeding. However, the value of any of these plants in practical control operations has yet to be demonstrated. It is of interest to note, however, that mosquito breeding is sometimes almost entirely eliminated in ponds that become completely covered with a thick growth of duckweeds (*Lemna*), which then acts as a mechanical deterrent.

The larvae of the principal malaria carrier (*A. quadrimaculatus* in the United States) are usually found most abundantly in water receiving more or less sunlight. This is correlated with their feeding habits, since their food consists of minute water organisms such as Chlorophyceae, which require sunlight for photosynthesis. In a discussion of illumination in relation to anopheline breeding, Boyd states that "many areas when covered by rank jungle are not of concern to the malariologist, but if cleared become of the utmost concern." This situation can be illustrated by an instance that occurred in the Federated Malay States where wholesale jungle clearing was said to have caused the disappearance of the shade-loving species, *A. umbrosus*, but provided excellent breeding places for *A. maculatus*, which is the more dangerous malaria carrier. The clearing of forest in Argentina is also credited with causing serious epidemics of malaria by making suitable breeding places for *A. albitarsis*, *A. tarsimaculatus*, and *A. argyritarsis*. Lumbering operations in southern United States have created similar conditions.

These instances emphasize the fact that, before malaria control operations are undertaken, the species of *Anopheles* concerned with the spread of malaria must be correctly determined in order that the control measures may be suitable and effective. Precautions must be taken, also, that situations are not created which will increase breeding opportunities for other more dangerous species of anophelines, or provide suitable breeding areas for those that may be introduced.

MALARIA AND IMPOUNDED WATER

Malaria control must be considered in the design, construction, and operation of any dams which create artificial ponds or lakes in the sections where malaria exists or may be introduced. At first glance the suggestion of a relationship between mosquitoes and dams may appear far-fetched; however, it is obvious that in building a dam and making an artificial pond, regardless of size, ideal breeding places may be created for the species of mosquitoes that transmit malaria. Provisions for malaria control during the construction and maintenance of impounded waters should be under governmental regulation.

In treatment of the subject, the term "impounded waters" refers to artificial bodies of water, large or small, with wholly or partly obstructed flow due to construction of dams. The words "ponds and lakes" will refer not to impounded areas but to natural bodies of water.

If it were not for impounded waters, ponds, swamps, and lakes, malaria would probably be a minor health problem in the United States, since such waters afford preferential breeding places for *Anopheles quadrimaculatus*, the principal, or probably the sole, natural vector of malaria in the Southeastern United States. However, the mere impounding of the water is not the only factor. Aquatic and semi-aquatic vegetation which pierce the water surface, dead vegetation, bark, and trash flottage on the water surface furnish the ideal breeding conditions for *Anopheles*.

The beginning of current knowledge of the influence of impounded waters upon malaria incidence were studies undertaken by H. R. Carter (18) and his associates in 1913. The first studies were made on the pond at Blewett's Falls, N. C., at Lock 12 on the Coosa River, and Lock 17 on the Warrior River in Alabama. Following the construction of a hydro-electric plant in Alabama, a great number of suits were filed against the company by nearby residents, claiming infection with malaria as a result of the creation of the lake. The court testimony showed that little was known of the problem at that time, but from 1914 to 1925 studies were made by the U. S. Public Health Service of a number of reservoirs in several States which formed the basis for all present State regulations governing impoundage of water. Regulations were offered as early as 1915, and in 1921 T. H. D. Griffiths (U. S. Public Health Service) drafted suggested regulations for the Federal Power Commission. The first State regulations were adopted by Alabama and Virginia in 1922. The Alabama regulations were declared illegal on a technicality, but were re-enacted in proper form in 1927.

The regulations are based upon eliminating, or making possible the control of conditions favorable to, the production of *Anopheles*, and these must be considered in the design and construction as well as in the operation of dams and reservoirs. Conditions favorable to *Anopheles* production are: (a) Constant water level, (b) collections of fine flottage or other débris, (c) aquatic or semi-aquatic vegetation which offers protection to the larvae, (d) minimum wave action, and (e) absence of natural enemies of mosquito larvae. The breeding ordinarily occurs in shallow water along the shore, especially in the upper ends of inlets or tributaries of the reservoir, and is most extensive during the early years of impounding before a biological balance is established.

Proper preparation of the basin is fundamental, and includes: (1) Clearing to provide a clean water surface and clean shore line; (2) draining areas which would retain water with fluctuations of the water level in the reservoir; and (3) provision for stocking with top-feeding minnows. The regulations governing the maintenance of a project following impoundage are directed primarily at biological control, which is not always practical, and must be supplemented by the use of larvicides. Considerable latitude in methods is permitted, since the regulations have been drawn with a view of the results rather than the means by which these results are to be obtained.

The design of the dam must be such as to provide for adequate fluctuation of water level, seasonal and periodic. By seasonal fluctuation is meant the maintaining of the water at the maximum normal level during the winter and spring, and lowering it to normal level at the beginning of the mosquito breeding

season. This inhibits the growth of vegetation, and when the water is lowered flottage will be stranded, with a resulting relatively clean shore line. Periodic fluctuation means the lowering of the water below the normal level at weekly or 10-day intervals during the mosquito breeding season—June, July, August, and September, and in some years, May and October. This fluctuation tends to discourage rank growth of aquatic and semi-aquatic vegetation. *Anopheles* larvae are stranded and die or are carried into open water where they may be destroyed by their natural enemies. With fluctuation, larvicidal measures are reduced, and in some cases may even be eliminated with consequent reduced costs. Without fluctuation, control by larvicides will be extremely expensive or even impossible. In storage reservoirs from which water is generally drawn during the dry summer months, the gradual draw-down serves much the same purpose as the periodic lowering and raising.

The construction schedule must be so planned that water is not impounded during the mosquito breeding months, for with the water rising slowly through uncleared areas, excessive production will result in spite of strenuous larvicidal measures, as was experienced on Lake Martin and Lake Wilson in Alabama. The optimum time of impounding is in the fall or winter so that there will be time for flottage to sink or to collect and become stranded on the shore.

In clearing the area to be inundated so that effective mosquito control can be maintained, it is necessary to remove all trees, under-growth, logs, etc., which may create or collect flottage between the maximum and minimum pool levels, to cut all trees and brush which would penetrate the surface and collect flottage at low water, and to remove vegetation along the shore line.

In the area between the high-water and low-water shore lines, the material is generally burned except for timber salvaged as lumber or fire wood. The same procedure is used in the shallow areas below the low-water shore line, but in deeper parts of the reservoir the trees which would penetrate the low-water surface are often felled and wired down to stumps. This is a less expensive operation, but unless well done may cause trouble by the trees breaking loose and floating to the surface, creating drift and flottage which interferes with malaria control operations, and also presents a hazard to navigation and to the operations of hydro-electric plants. Additional clearing and brushing is necessary beyond the high-water shore line in order to provide as clear a shore as possible.

Fluctuation in water level alone is not always sufficient to control *Anopheles* production and other provisions for malaria control should always be included. Screening of homes within flight range (at least 1 mile) of the impounded body of water should be standard procedure—in fact the average standard of living of the American people to-day requires a screened home for every family, so this cannot be termed an extra measure of precaution on impounded waters. Applications of larvicide to supplement proper impounding procedure, wherever and whenever necessary, have been adopted by hydro-electric companies on the recommendations of health authorities.

There are two generally accepted larvicides, namely, various mixtures of oil, consisting chiefly of petroleum products, and Paris green "dust." Oil was first used and still has a wider application because of its toxicity to a greater

variety of mosquito larvae. The equipment for applying larvicide on impounded waters has had progressive development since the work was first undertaken, before 1928.

Oil was first applied through various hand-operated spray pumps mounted on boats paddled by hand or powered by inboard motors. Then pneumatic spray tanks came into use, operating with compressed air; they were first used from paddled boats, and later from boats with outboard motors which enabled the operators to maneuver their way in and out of the most difficult places with ease.

The next development was the "water oil" method of applying oil which has had wide adoption on the larger projects. In this method oil is carried from boat to object by a water stream. A small motor-driven centrifugal pump taking suction from the lake provides the water stream. Oil from the storage tanks is introduced into the suction of the pump through a small pipe. A valve on this line enables the operator to vary the amount of oil discharged at will. Initially the boats carrying this equipment were powered by outboard motors although, recently, inboard motor-driven units have been provided. The one inboard motor serves to operate the pump as well as to propel the boat.

The oil and water stream of the "water-oil" unit can be made to carry 60 to 100 ft which is farther than that of the pneumatic oil sprayers. The stream will also carry well in the face of a breeze. The force of the stream can be made to break up accumulated flottage and carry through heavy vegetation to protected breeding places. The range tends to permit keeping the boat in deeper water, thus reducing damage to the propeller shaft.

Several years ago the use of Paris green was given new impetus and was applied to some extent on impounded water. In this work it has been applied by both power and hand-driven dusters mounted in paddle or motor-driven boats. Various inert diluents and mixtures of Paris green have been used. A popular one is 5% Paris green to 95% hydrated lime by volume. The Paris green must be of a good grade containing at least 50% arsenious oxide and of known toxicity for larvae. The rate of application is from $\frac{2}{3}$ to 1 lb of Paris green per acre.

Occasionally areas are found on lakes where the use of hand spraying or dusting equipment is necessary. A number of commercial apparatuses are on the market. Those most commonly used are the knapsack and pneumatic sprayers and hand dusters. Although larvicides will continue to be used, the present (1939) tendency is to limit this temporary work and spend available money on cleaning regrowth from the shore line, for more lasting effects.

After water has been impounded it is the practice to keep the lake under careful observation by making periodic inspections for larvae and adult mosquitoes. A trained and experienced field man is necessary. He is usually provided by the company and co-operates with health officials, submitting weekly reports covering the results of observations and control operations. Various sample reports and permit forms are shown in Appendix II.

In making inspections, the extent of mosquito breeding is determined by dipping along the shore line among vegetation and flottage, and recording the number and stage of development of the mosquito larvae. Inspections are also

made for adult *Anopheles* to check the effectiveness of the control work. The malaria-carrying mosquito rests during the daytime, preferably in a dark sheltered spot. Mosquito-catching stations are established at intervals around the reservoir at residences, barns, under-side of bridges, etc., which are examined at regular periods with the aid of a flash-light, and the mosquitoes caught are identified.

For intelligent planning and operation of malaria control measures, it is essential to know the distribution of population, together with the prevalence and distribution of malaria in the area affected, so that work can be intensified in the more important areas or efforts reduced where the problem is less acute. It is also important to determine the malaria incidence before, and from time to time after, impoundage so that the effect of impoundage and effectiveness of control can be measured. The population affected is that living within 1 mile of the reservoir, the approximate flight range of the malaria-carrying mosquito.

The completion of major hydro-electric projects often requires many months or perhaps several years. During the construction period hundreds of workmen, as well as many of their families, must be housed in semi-permanent or temporary camps. Living environments must be pleasant and health conditions maintained above the possibility of any serious disease outbreak. The medical staff in charge must be active indeed to safeguard the lives and health of these people.

Workmen will be drawn from many sources and some may come from malarious areas. There is then the probability that infected persons may enter the camps at any time. Therefore, prevention measures in the groups are important. Control of the vector in and about semi-permanent camps in the summer is feasible through ditching and larvicidal operations. These primary measures may not be feasible in protecting the workmen housed in temporary camps. In such cases resort must be made to the less protective so-called "secondary measures" of screening, use of bed nets, insecticides, etc.

It is now the general policy of hydro-electric companies in the South to spare no effort toward preventing the introduction and spread of malaria on and about their projects. A careful check is kept on the inhabitants, particularly new arrivals, and where the disease is found, prompt corrective measures are used. This is done not alone in an effort to keep construction labor in a healthy condition but to have as few possible sources of infection later when the lake is impounded.

It may be stated that there should be no conflict whatever between hydro-electric development and health conservation. Rather, the two together are workers for the common good; water power promoting industrial and social progress, and public health providing an atmosphere in which enterprise can live and move.

MALARIA AND TRANSPORTATION

Malaria was introduced into the United States by European emigrants to the Atlantic Seaboard, and its spread to the West Coast followed the pioneers

in their westward movement and colonization along navigable streams used for transportation. Malaria had already become deep-rooted before the era of railroad construction in the United States. The vast river basins emptying into the Atlantic Ocean and the Gulf of Mexico, with the millions of fertile acres ready for the axe and plow, needed only the human malaria host to become the endemic foci of malaria; and, although there has been a recession of malaria from the northern habitat of the disease, and perhaps a lessening in the severity of malaria attacks over the entire area, it is doubtful if there has been any decrease in the possibilities for active transmission, even with the high state of cultivation of land. Topography, climate, and susceptible people have maintained and will maintain the foci of malaria in its present widespread distribution, with continual expansion to newly occupied lands through immigration, until the disease is attacked on the basis of areas, whether river basins or sections of States, rather than on the community or county-wide basis as at present.

Railroads.—Railroads have a common interest with the communities they serve. This mutual interest is intensified in regions in which malaria is a serious public health problem, and it is undoubtedly true that no other preventable disease affects railroad operation in the South so adversely as malaria. Sanitation on the railroads began with their construction. The history of pioneer railroad building is one of fighting disease, as well as of extending rails through malaria swamps and over mountains. Officials of certain railroad companies recognize the excessively high cost of location and construction engineering in their valuation briefs, due to the devastating effect of malaria on engineers and construction laborers. Even now the disease saps the vital energy of employees, handicaps industrial activities in communities along the railroads, and renders untold acres of fertile land unprofitable, if not actually uninhabitable.

Railroad labor for construction and maintenance in the Southwest is recruited largely from people living along the railroads on farms or in the smaller towns, to which a floating Mexican labor population is added from time to time. Since malaria is a rural rather than an urban disease, one can expect a considerable proportion of these laborers to be infected. The railroads and their employees share the burden of malaria. The railroads must hire and pay for inefficient labor, and the employees and their dependents must suffer the agonies of "chills and fever" and the cost of medical care as well as the loss of wages. A high malaria rate means an excessive turn-over of labor and a very definite loss through salaried officers and employees during illness. Infected men working with machinery or on moving trains are fit subjects for accidents. In the maintenance-of-way departments of railroad companies, construction activities are frequently retarded during the malaria season. Expensive equipment is sometimes tied up due to lowered efficiency, or frequently through actual deficiency of labor. One observer found one-third of a large construction extra gang either on hospital sick-leave for malaria, or "chilling" in their bunks at the time of his visit. In order to have sufficient number of workers in this instance, it had become necessary to employ 50% more men than required, and

the railroad company was obliged to provide 50% greater housing facilities and tools.

A high malaria rate means an excessive turn-over of labor and a definite cost for replacing labor; but by far the greatest cost of malaria to railroads is due to the hundreds and thousands of employees who continue to work for weeks or months while infected, and who can return but feeble efforts for their wages. The seriousness of this problem of malaria among railroad laborers in the South is reflected by a study of railway hospital records, which show that fully one-third of all hospital medical cases were chargeable to this disease alone. This rate was based on an analysis of records covering a period of 20 yr, ending in 1917, and has been found fairly constant for other railway hospitals since that time. In addition to the high percentage of malaria cases in the hospitals, further information reveals that from two to five times as many employees receive temporary relief from malaria through local physicians or other agencies, other than hospitals. The same degree of malaria prevalence among the employees will necessarily be reflected in the rates among their dependents. It is obvious that the railroads carry a much greater indirect burden from the disease in the loss of business due to the lowering of output from farm and factory throughout the high malaria districts.

The problem of protecting railway employees from malaria infection is complicated because of the various conditions under which certain of them live and work. Men employed in bridge and building departments, on extra gangs, or other mobile forces, live in camp cars that are moved as necessary from point to point over their territory. These men spend much of their time in river bottoms, where they necessarily are exposed to hordes of malaria mosquitoes. These employees, comprising on an average about 6% of total railroad employees, furnish about 35% of the railroad hospital malaria cases. Employees in train service frequently tie up at terminals in small communities without adequate screening of their sleeping quarters. Station employees, shopmen, switchmen, and watchmen are employed on day and night shifts, and in the more remote stations are constantly exposed to mosquito infection. Some of the larger shop terminals are located adjacent to swamp areas that are directly responsible for enormous charges because of inefficient shop operation. Shopmen will comprise about 30% of all employees and will furnish about 12% of the hospital malaria cases.

In normal times, roughly 20% of a railroad company's employees will be section laborers, and these men are almost wholly recruited from small communities or rural districts where the malaria rate is high. This group of employees has usually furnished more than 40% of the hospital malaria cases.

The railroads themselves have been responsible for the spread of some malaria, particularly in the impounding of water for water supply purposes and the discharge of waste water from terminals without adequate drainage. The improper placing of culverts and failure to enforce complete drainage of borrow-pits are also a serious malaria menace. These sins of commission, however, are but "a drop in the bucket" compared with the effects on malaria of the numerous construction activities and the natural possibilities for mosquito propagation in the southern territories. However, mistakes made in the past

during railroad construction need not be repeated now that engineers have become enlightened regarding the cause of malaria. This should apply with equal force to engineers engaged in all kinds of construction projects in the South. The late H. R. Carter, a world authority on malaria, stated that more than 50% of the malaria cases in many parts of this country are "man-made" (50).

New "man-made" malaria conditions will continue to be created as long as Federal, State, and local health authorities acquiesce. Many of these individuals who plan construction projects may be uninformed as to the creation of malaria hazards, but there is no excuse for health authorities who have ample information and usually the power to prevent the building of new malaria mosquito propagation areas.

Any plan for the control of malaria among railway employees should be based on obtaining the maximum protection for the least cost, and every known method can be applied effectively. For the protection of men employed in large shops, it will be found most expedient to institute an anopheline control campaign through drainage, oiling, and filling, and other field means, since this would be similar in all respects to a small town control project. For extra gangs of bridge and building departments, men living in cars moved from place to place on the road, the first defense will be the proper screening of their sleeping and eating quarters with 16-mesh wire, the swatting of mosquitoes in the cars, the location of outfit cars wherever possible in localities free from malaria mosquitoes, and medical treatment of infected individuals. For section men living in isolated houses, screening will be the first line of defense. It is sometimes possible to eliminate mosquito breeding in the areas adjacent to section houses. The foregoing three classes of employees comprise approximately 50% of the total employees on a railroad, and will furnish more than 85% of the hospital malaria cases.

Permanent, or semi-permanent, engineering construction camps can be built and maintained so as to give full protection from malaria mosquitoes to those housed therein. The first consideration should be that of choosing a camp site affording as much freedom as possible from mosquitoes. Adequate 16-mesh screening should be provided for sleeping, cooking, and dining quarters, with double-door entrances, and covers for chimneys not in use.

Temporary protection can be effected by use of a bed net or a mosquito net tent fly. Any attempt to provide protection from malaria mosquitoes will be futile unless those who are in charge of laborers will explain the precautions to be taken and the reasons for maintaining the screening or nets in good condition. Frequent inspection of quarters, and swatting of mosquitoes found, will be the only means of appraising the effectiveness of screening, so these should be routine items on the part of those in direct charge of camps. Men should be taught to look over their clothing, including their hats, before entering quarters.

Immediate isolation or hospitalization of suspected malaria cases should be the rule. Removal of standing water within the camp area, and oiling of water surfaces producing mosquitoes, can be practiced successfully, just as in any small rural area.

Highways.—The great impetus to the building of highways through southern States has augmented the malaria problem, both directly and indirectly—directly by inadequate drainage of borrow-pits, by placing of culverts at too high an elevation for complete drainage with the consequent impounding of water, and by the creating of gravel and clay pits from which road materials were taken; indirectly, the possibility for people using the roads for recreation during the early evening has introduced a new malaria hazard in that these motorists are constantly exposed to malaria infection while driving through rural districts. The building of road houses and tourist camps has also added to the possibilities for malaria infection, inasmuch as many of these camps are not adequately screened to protect their occupants, and many of them have no screens. In addition, they are generally located along some body of water which furnishes an attractive camp.

A malaria worker in Texas has reported that the construction of good roads in the western half of the State, which heretofore has been thought to be outside of the malaria district, has resulted in the local spread of malaria by creating new breeding areas because of the fact that settlers from malarious areas follow the building up of communities along these good roads. Investigators are constantly finding evidence of the spread of malaria following the building of good roads in the Southern States, and this has been particularly true in areas that are being developed for agricultural or horticultural purposes.

During new road construction the profile of the road-side ditches should be quite similar to the center-line profile of the highway. Adequate outlet drains and culverts "to grade" should always precede the construction of a "fill." Often a location engineer will fail to provide culvert facilities to care for storm water on a drainage area, or he may place the culvert too high to provide for removal of residual water. Consequently, engineers interested in malaria control have often found it necessary later to find some means for forcing a pipe culvert through the fill to provide for such drainage. This method is necessary to avoid interrupting traffic, but the cost is exorbitant as compared with proper original construction. A culvert placed too high often necessitates cutting through the concrete invert or the lowering of the pipe in order to establish an effective grade. Close adherence to these procedures is not only followed by reduced road maintenance costs but has been followed frequently by a material reduction of the incidence of malaria in the farms and in the villages served by the road.

Airplanes.—Since the U. S. Public Health Service has conducted research in the transportation of mosquitoes by airplanes, the potential importance of the spread of yellow fever, through the carrying of *Aedes aegypti* and infected man, has been emphasized. In addition to yellow fever there is also the possibility of the introduction of other and new species into new territory. Already it is thought probable by some that *Anopheles gambiae*, an important vector of malaria in its native habitat, West Africa, was introduced into Brazil by airplanes. Students of yellow fever see the possibility of future devastating epidemics of yellow fever over wide areas where the disease has never been known before, through chance introduction of infected *Aedes aegypti*, or, as a conse-

quence of rapid travel, the carrying of infected persons into new areas with large non-immune populations.

Avenues of travel—the railroads, the navigable streams, the modern highways for automobiles and trucks, and the airplanes—have facilitated the movement of people, who have been the instruments for introducing malaria, or of re-introducing it, in new and unsuspected places. The changes in the industrial situation in recent years have given impetus to the spread of malaria. Among these may be mentioned the movement of a large negro population to the North and East and the movement of southern lumbermen and employees to the Northwest.

From the foregoing, one can readily visualize a national malaria problem, and one demanding national planning and execution with the full co-operation of all the affected States. The remarkable success in the reduction of malaria obtained by some railroad companies—namely the St. Louis Southwestern, the Missouri Pacific, and the Rock Island, following the pioneering work of the St. Louis Southwestern in 1917—have stimulated similar measures among many southern corporations. This has been the means of starting municipal mosquito control campaigns for the benefit of a large proportion of the southern urban population. Malaria is still firmly embedded, as firmly as in 1916 when the malaria campaigns were started in Crossett and Lake Village, Ark., and it appears as if it would remain so until attacked on a national scale and in such manner as will eradicate the thousands of endemic foci from the “byways” and “forks of the creek.”

THE CONTROL OF MALARIA

Thus far, this report has dealt with malaria as a problem in the United States. The factors entering into the transmission of the disease by *A. quadrimaculatus* mosquitoes have been treated at length. The specialized problem and measures employed for control of malaria on artificially impounded water have been given special attention. Next to be considered are control measures generally applicable to the problem in this country. State and County Health Departments are the organizations normally concerned with the control of malaria. Within these organizations there are engineers, or men working under engineering direction, who are charged with the execution of measures aimed at malaria control. There will also be found the malariologist, the epidemiologist, and the vital statistician whose work is essential to the analysis of the problem and the planning and execution of a control program.

Location of the Problem.—The first step in any malaria control program should be the assembly of facts and information bearing on the location of areas where the disease is a problem. Death and morbidity statistics are drawn upon or collected by surveys to obtain relative incidence of the disease, first by States through counties, and then subdivisions or sections of counties.

The location of the problem appears most important when the statement is made that malaria seldom, if ever, occurs uniformly over a given territory. The several types of surveys are listed as follows: (a) Blood surveys, (b) spleen surveys, (c) history surveys, and (d) anopheline surveys. Each has its merits.

One, or a combination, is used depending on the information sought and the means or persons available for doing the work.

The results of statistical study and surveys are usually plotted on maps to show foci of infection. In addition these maps should convey the results of hydrographic surveys of ponded areas, creeks, natural and artificial drains, and entomological surveys of anopheline production.

In developing a malaria control program in any area, the fundamental principles on which malaria control is based must first be understood. The general methods of attack against malaria are to: (1) Control production of anopheline mosquitoes; (2) secure proper treatment of infected persons by physicians; and (3) place a barrier between Man and anopheline mosquitoes.

The first method, the one which has been used with much success in the past in the Southern States, is referred to as "Mosquito Control." The second is accomplished through proper treatment under the care of a licensed physician. The third, which is being used on a larger scale than ever before, is referred to as screening or mosquito-proofing the houses.

Where the ratio of density of population, or of wealth or income per acre, to the cost of complete mosquito control measures is high, then complete mosquito control work will be the most successful and generally applicable method. The number of measures possible decreases rapidly as this ratio diminishes and it becomes necessary to use whatever effective measures remain possible with the financial support available. There comes a time when the public money available for public or county-wide work is zero, and the ratio of density of population, or of wealth, to cost of control is low. A very effective measure is still left and that is the placing of a barrier between Man and the mosquito by mosquito-proofing the houses at the expense of the farm owners, tenants, or individual owners.

Anopheline Control Measures.—The weak link in the life cycle of the mosquito is the larval stage. Destroying adult mosquitoes after they have emerged from their breeding places has not been successful except in places where they can be enclosed. Mosquito eggs may withstand desiccation or freezing and still hatch when covered with water. Being aquatic in the larval and pupal stages, it is during this period that anti-mosquito operations can be effective. The attack may consist of: (a) Drying up the places in which larvae and pupae may develop; or (b) making such places unsuited for their growth. The first includes drainage of anopheline mosquito breeding waters, or filling low areas where water may collect. The second includes: (a) Application of larvicides including oil and Paris green; and (b) removal of protective vegetation and flotation to facilitate the activities of insect enemies.

Drainage.—Where it is possible, economically and otherwise, drainage is the most effective permanent method of reducing or controlling malaria. It is advisable, therefore, to stress its importance and to outline a few of the general fundamental principles involved.

Agricultural Drainage.—Considerable drainage work has been done for agricultural purposes by organized drainage districts. However, one should bear in mind that agricultural drainage and malaria drainage are very often extremely divergent in both plan and purpose. Agricultural drainage not only

removes water from the surface, but lowers the water table; malaria drainage removes standing water from the surface. Usually, too little attention is given to the avoidance of pockets and pot-holes in the bottoms of agricultural drainage ditches, so that, although the fields may drain perfectly, a new series of breeding places are created in the ditches themselves. It is a fact, however, that agricultural drainage often reduces malaria to some extent and malaria drainage certainly benefits agriculture; however, each problem is too scientifically precise in itself for practical co-ordination.

Undrained Borrow-Pits.—It is not uncommon for persons engaged in highway and railroad construction to leave one or a series of undrained borrow-pits along the roadway. Such places become intermittent or permanent producing areas for anopheline mosquitoes. Whenever possible, all such borrow-pits should be self-draining at the time of low water. Many miles of these borrow-pits and wide-bottom ditches that become a series of pools, unfortunately, are being made on construction work under the supervision of engineers.

Culverts.—Culverts along railroads and highways are frequently placed too high. This causes pools to form and often renders impossible subsequent drainage of standing water up stream from the culvert. This condition is sometimes corrected by installing an additional culvert, a smaller pipe at the correct grade to carry off residual water.

In a malarious territory, pipe culverts or concrete-bottom bridges should be located only after consideration has been given to the possible future drainage of any ponded area that may exist up stream. The grade of the culvert invert should be straight, and no pockets or pool-shaped sections should be allowed. The cement used at the joints should not be allowed to ooze up on the inside, forming a dam and pools of water, but these joints should be smooth before considering the job finished. In small drains (say, 12-in. to 15-in. pipe) it is difficult to correct this condition after construction without re-laying the pipe.

The floor or invert of all culverts should have a circular cross-section with an ordinate distance of, say, one-sixth of the chord of the curve, to provide for velocity sufficient during the dry-weather flow to prevent mosquito breeding. A flat, level bottom allows pools to form with little or no velocity through them and mosquitoes can, and do, breed in such places.

Concrete Aprons.—Where water flows rapidly through culverts, deep pot-holes are often formed at the outlet ends. These pot-holes, in many instances, hold water the year round and constitute favorable breeding places for anopheline mosquitoes. Filled with brickbats, native stone, broken concrete, or any other durable material these pot-holes may be brought to grade and a concrete apron constructed, thereby permanently eliminating such breeding places. The cost of such aprons is usually little more than the cost of the cement, and they constitute an important part of the malaria control program.

Clay-Pits and Other Excavations.—Pits from which clay is obtained for the manufacture of brick, pottery, and tile are seldom properly drained and frequently are located within *Anopheles* flight range of towns and villages. Such pits, as well as quarries and other excavations, ultimately become permanent sources of malaria mosquitoes.

Faulty Drainage Construction.—Drainage measures are often referred to as "health measures" but frequently fail in that respect because more attention is devoted to rapid removal of storm water than to the removal of all water after the storm has passed. It is the latter water only that is a menace to health. Frequently, pools occur near embankments which could have been drained at little cost before the embankments were made, but will be expensive to drain later. In firm soils, when wide ditches are necessary, it is sometimes advantageous to have a small ditch within the bed and in the center of the wide-bottom ditch. For malaria control purposes, it is advisable to eliminate sluggish or standing water, or to reduce the water surface to a minimum. Arrangements should be made for a regular maintenance of ditching systems. Where possible, ditches should have clean-cut, sloping sides, narrow bottoms, even grades, and straight courses. Ditches with permanent flow should have a minimum number of branch or spur ditches. Branch ditches should join the main ditch at an acute angle or curve in order to lessen deposition of sediment in time of storm. The bottom of the two ditches at their junction should be at the same elevation. In order to make this arrangement, the grade of the lateral ditch may have to be increased in the last 50 to 300 ft. This will prevent scouring on bank and bottom of main ditch by water from the lateral. Where ditching is done without regard to mosquito control it often becomes a prolific source of malaria mosquitoes.

Irrigation Projects.—In some of the irrigation systems water seeps through the embankments into depressions and undrained borrow-pits. Irrigation projects should always be accompanied by drainage projects, which are essential from both agricultural and malaria standpoints.

Faulty Street and Roadside Ditches.—Some communities have found it advantageous to employ civil engineers as city managers, and it is important that such officials, as well as city and county engineers, realize the relation of malaria control to the welfare and development of the community they represent. Street ditches that may retain storm water should be kept at proper grade and should be given more attention and inspection than those that remain dry. As a rule this is not done and as a result the taxpayer, in addition to paying for road maintenance, may have his earning power decreased and a doctor and drug bill to pay for curing the disease caused by neglected ditches.

Concrete-Lined Ditches.—When streams cannot be carried underground through communities, their bottom section should be lined with concrete or some other suitable material. In many instances, it is advisable to make this concrete-lined section conform to the invert of the future storm-water culvert. This narrow central section will keep the water in motion and will prevent anopheline propagation.

There is a great opportunity for the engineer to give advice regarding the municipal activities that will eliminate parasite-bearing and obnoxious mosquitoes. The permanent results obtainable by economical concrete lining of ditches are such that the entire public can see and appreciate.

Catch Basins.—Storm-water catch basins are frequently a prolific source of non-anopheline mosquitoes, and when possible should be modified so as to

be self-draining during dry periods as part of a general mosquito control program.

Filling of Ponds.—Many natural depressions lying within towns and cities, where drainage is impractical, may be economically eliminated by filling with trash, street sweepings, and other non-putrescible débris, with a topping of earth.

Street Extensions.—The extensions of streets in suburbs on flat and water-tight lands where the street ditches will hold water may be a serious menace to health. It should be the business of the engineer to see that street ditches are properly drained and, if possible, made permanent by the installation of concrete curbs and gutters.

Malaria Control Drainage Practice.—In thickly settled communities the cost of drainage operations for *Anopheles* control and for its maintenance should be determined before the work is started. Drainage for malaria control consists of the removal of water that remains after rain storms have passed. It is not essential that the remaining storm-water be removed rapidly but it is advisable to keep the water in permanently flowing ditches in fairly rapid motion. For that reason it is advantageous for the ditches to be constructed with straight courses, side-walls sloped according to the type of soil, and with narrow V-bottom cross-section. Permanent pools and swampy areas should be drained, cleaned, or so reduced in area as to lessen subsequent treatment.

In soggy soils there is a tendency to install too many branch or spur ditches. This increases the cost of maintenance considerably. Therefore, it is advisable in draining swampy areas for malaria control purposes to install main ditches and principal laterals and then give the ground an opportunity to dry out before locating the small ditches and spurs. Generally, less ditching is required for mosquito control than would be required for agricultural purposes.

Ditch Maintenance.—So far as mosquito production is concerned, the maintenance of ditches in proper condition is often almost as important as the original ditching. It is necessary to keep that part of the ditch below the flow line free of obstruction, débris, and vegetation.

Oiling.—Petroleum oil is one of the principal larvicides used to-day in the control of mosquito breeding areas which cannot be drained, filled, or otherwise controlled. A thin film of oil is applied to the water surface every week during the warmer months, or 10 days at the beginning or end of the season. In the two active aquatic stages (that is, the larval and pupal stages) the oil poisons the system by entering through the breathing tubes. The more volatile the oil, the quicker the killing time. Oil does not mix with the water but floats as a film on the surface, and as applied for mosquito control is not injurious to fish, but may discolor and damage vegetation. Oil can be applied in many ways, the principal equipment used being various types of knapsack or hand sprayers and mechanical devices mounted in boats or on conveyers. In general the quantity of oil (3 parts kerosene to 1 part black oil) to be used will vary from 3 to 25 gal per acre, depending upon the method of application and the amount of vegetation, flottage, and débris. The use of oil as a larvicide has some disadvantages. It is messy and bulky to handle and transport; it cannot ordinarily be applied at a distance from the sprayer; it spreads poorly in vegeta-

tion or débris and is very destructive to the rubber hose on the sprayer. Some oils are too heavy to form an even film over the entire water surface, others are so volatile that the film is scarcely noticeable on the surface and the oil evaporates very quickly, disappearing before all of the larvae and pupae have been killed. The spreading power of oils can be greatly increased by diluting with 2% of crude castor oil. Light distillate oils are efficient and most practical in actual control work.

Larvicides.—Paris green diluted with inert dust such as hydrated lime so that the mixture will contain from 1% to 50% of Paris green is used as a dust larvicide. Roughly, this dilution may be classified as follows: (a) For hand broadcasting at distances up to 20 ft, 1% to 2% mixture; (b) for use with hand-operated dustgun, up to 200 ft, 5% to 10% mixture; and (c) for use in dusting by airplane, 5% to 50% mixture. Inasmuch as the Paris green remains on the surface of the water it is most effective against the surface feeding anopheline larvae. As ordinarily used in mosquito control work, Paris green is not effective against other species of mosquitoes; nor does it injure any other form of water life.

The Paris green dust must be thoroughly mixed with the diluent. It is applied by hand or by an ordinary plant or cotton duster from the windward side of the water deposit. Advantage is taken of the wind to carry it across the area. Therefore, it requires intelligence and "a sharp eye" to watch where the cloud goes. A water deposit is well dusted when a dust cloud of moderate density is observed to pass over the area.

It must be remembered that Paris green is a poison; hence care must be exercised that no large clumps of dust are dropped from the blowers where persons or live stock might get to them. The dust itself so well distributes the small amount of Paris green that there is no danger of poisoning fish or live-stock even when dust is visible on the vegetation. In fact, Paris green has been used for years on water deposits where cattle drink without the development of any signs of poisoning.

The advantages of Paris green dust as a larvicide are as follows:

- (1) It may be applied to areas for controlling or limiting anopheline mosquito breeding where other larvicides are not practical of application;
- (2) Advantage can be taken of favorable air currents to carry the dust cloud over a wide area, otherwise inaccessible;
- (3) Areas covered with emergent vegetation are effectively treated by this method; and,
- (4) It is easy to transport.

The main disadvantage of Paris green, of course, is the fact that, when applied to the water surface, only *Anopheles* larvae in the second, third, and fourth stages of development are killed. A part of the first-stage larvae escape, as well as all pupae.

Pyrethrum, or Persian insect powder, is in itself a poor larvicide, but recently the active principle has been extracted with kerosene and then emulsified with soap solution. Diluted, this mixture forms an efficient fluid larvicide against all species of mosquitoes. It is applied as a spray. The effectiveness

of this larvicide depends on the care with which the emulsion is made, and special mixing apparatus is essential to form a good emulsion. When properly applied it will not harm or discolor foliage or vegetation; nor is it injurious to fish life and hence it is ideal for ornamental lily ponds.

Minnows.—*Gambusia affinis* (commonly called the top-feeding minnow) and other species of minnows have been successfully used for mosquito control in ponds, barrels, cisterns, wells, etc. *Gambusia* multiply very rapidly, giving birth to their young in successive broods throughout the warm season. They are top-feeders and are exceedingly fond of mosquito larvae. The surface of the water must be free of flottage, vegetation, algae, and other hiding places of the mosquito larvae in order to give the top-minnows access to the larvae over the entire surface.

Screening and Mosquito-Proofing.—If mosquitoes are prevented from biting malaria carriers, it is easy to understand how they are harmless even if they do bite well people. Likewise, if malaria-infected mosquitoes are prevented from biting well people they will not spread malaria. Good screening as a community achievement accomplishes both of these objects and hence strikes a doubly effective blow at mosquito-borne diseases. Good screening is of further advantage in public health because it reduces, decidedly, the prevalence of fly-carried summer complaints of children, with their high death rates.

A properly screened house is one in which all openings are protected against the entrance of mosquitoes through the effective use of 16-mesh screen over doors and windows, and sealing over apertures. A well-screened house, therefore, means good screening combined with effective mosquito-proofing. In general, screening is indicated as a malaria control measure where dwellings are scattered and mosquito breeding waters would be costly to eliminate or control.

Pyrethrum spray is often used inside of houses to kill any mosquitoes which may have gained access.

In designing houses for location in malarious areas due attention should be given to permanent mosquito proofing. Durable screen cloth, 16-mesh per in., should be used on doors and windows. Heavy, substantial screen doors with automatic closing devices should be used and should always open outward. The flooring should be of tongue-and-groove material. Walls, ceiling, and floors should be sealed with a substantial material.

It is frequently necessary to locate construction camps in malarious territory. If possible, the camps should be placed a mile or more from any water areas suitable for *Anopheles* mosquito production. If not, the drainage or control of the area within *Anopheles* flight range should be planned as a part of the camp protection and quarters should be mosquito-proofed.

DRAINAGE FOR MALARIA CONTROL

Drainage is the most effective permanent method of reducing or controlling malaria. Simple ditch construction is not synonymous with drainage, as the term "drainage for malaria control" embraces the proper planning, construction, and maintenance of ditches, streams, canals, etc., and all appurtenances

thereto, for the control of anopheline mosquito breeding. This is specialized work which should be designed and executed whenever possible by experienced personnel. If the engineer in charge has not had experience in this particular field, he should be guided by the recommendations of those who are so qualified.

Valuable land is often reclaimed by malaria-control drainage, but questionable agricultural drainage programs should not be undertaken under the guise of malaria control. Competent drainage engineers, inexperienced in this work, are often so engrossed in the apparently larger phases of the work that the finer points so essential to successful malaria control are overlooked.

Over-drainage may be quite as objectionable to the best interests of a community as the lack of drainage, and one must study the need diligently before proceeding with a ditching program.

RECONNAISSANCE SURVEY

Malaria seldom, if ever, occurs uniformly over a given territory. Before attacking a drainage program the engineer should make preliminary investigations, including the following factors: (a) Location of breeding areas; (b) character of breeding areas; (c) number of people living within 1 mile of such breeding areas; (d) number of people giving history of having had malaria; (e) type of soil, timber, and vegetation; and (f) means whereby maintenance will be obtained if the drainage works are constructed.

The location of breeding areas should include actual and potential breeding areas, permanent and temporary water. Surveys made during dry periods sometimes record few breeding areas, whereas those made in a wet season may develop an entirely different situation. The observations on a single survey can rarely be depended on for the planning of a complete program.

The character of the breeding area will determine what measures must be taken for control work. The type and amount of breeding must be noted. This will include the examination of mosquito larvae, and also the catching and identification of adult mosquitoes in the immediate vicinity.

The number of people living within 1 mile of the breeding areas and the cost of ditching will determine whether a ditching program would be economically sound. Political boundaries must not be used in establishing a protective zone. A dense population may live beyond city boundaries, or there may be large areas within city limits having a scattered population which would not justify expensive control measures. In any case, mosquitoes know nothing of artificial boundaries, and the most effective barrier is a mile or more of distance between the people who are to be protected and the nearest breeding area. Where unusually heavy mosquito breeding is found it may be necessary to extend the control area beyond this theoretical mile limit.

A check on the number of people giving a history of having had malaria is helpful many times in determining whether control measures are practicable. The type of soil, timber, and vegetation through which a proposed system would be constructed will be useful in determining the cross-section,* the carrying capacity, and the cost of the ditches. Maintenance of the system is more important in malaria control than in agricultural drainage. Poor main-

tenance often nullifies the results of a drainage program. The feasibility of doing "permanent" work should always be considered.

Existing maps are helpful in orienting one's self in a new territory. Using only this as a guide, the engineer will prepare a new chart of actual findings, including drainage channels and all appurtenances, actual and potential breeding areas, mosquito catching stations, distribution of population, malaria history data, topography and soil formation, and finally, the probable necessary drainage works required, including the ultimate outfall.

With this information one experienced in malaria control may determine whether a drainage program is desirable, feasible, or economically sound.

DITCHING SPECIFICATIONS

Main ditches and canals are always installed first, and, whenever possible, some time should elapse before laterals are constructed. It is often found that fewer laterals will suffice if the full benefits from the main ditches are observed first. Every foot of unnecessary ditch eliminated means that much less maintenance in the future.

Location.—Preliminary levels should always be run on controversial lines before a selection is made for a new ditch, and the outlet into which it discharges must care amply for all of the water that may be delivered to it.

Ditches should be made as straight as possible, and all changes in direction should be made with curves as flat as the contour of the ground will permit. Sometimes it is necessary to follow property lines in order to avoid cutting across small parcels of land, but sharp bends in ditches are to be avoided as they are unsatisfactory and difficult to maintain. Lines located in natural depressions are usually the most satisfactory as the ditches need not be so deep and the necessity for other ditches through such areas is eliminated.

By selecting the location of earth ditches carefully there will be no need of change when permanent linings or channels are built. This is especially important in heavy construction work.

Substantial stakes should be set on all permanent location lines, with markings as simple as possible, because the labor foreman in charge of construction is usually inexperienced in malaria control ditching.

Cross-Section.—The size of the ditch is dependent on slope, type of soil, wooded area, and rainfall. The cross-section should be just large enough to drain a given area within a week or less, and carry the normal run-off water in the ditch section.

Narrow-bottom ditches are usually adopted in malaria control work as they do not fill up as rapidly as flat, wide-bottom ditches. The flow of water is more rapid in narrow-bottom ditches so that mosquito larvae are more easily washed away. A V-shaped bottom is ideal for malaria control ditching because it constricts the water area to the greatest extent.

The section must be designed to prevent undue erosion, and allowance should be made for weathering of the side slope. In average soils loose material soon slides into the ditch and reduces the bottom width. Cracks caused by drying of the soil, or freezing and thawing of the soil, loosen material on the side

slopes. This loose material falls into the ditch, reducing bottom width, and may destroy the grade of the ditch.

When possible the ditch cross-section should conform to the principle shown in Fig. 6. The dimensions shown are for a 2-ft bottom. The ditch is first built to grade on the line AB , and then finished to the shape shown (ACB)—a very flat V. A slope of 2 in. to 1 ft from each side to the center of the ditch is provided; thus, a 2-ft bottom will drop 2 in. in the center, and a 4-ft ditch will drop 4 in. For smaller ditches, finishing with a round-pointed shovel will shape the ditch nicely. The bottom shape of ditches often represents the difference between success and failure of malaria control work.

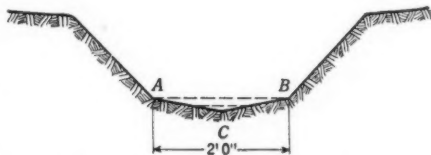


FIG. 6.—V-SHAPED DRAINAGE DITCH

Sometimes malaria control and agricultural drainage can be co-ordinated by increasing the bottom-width of malaria control ditches to permit rapid run-off from prospective farm lands. This procedure may help solve the difficult maintenance problem, as property owners often agree to maintain the ditches in consideration of the aid given in primary construction.

The side-slope of the ditch under average conditions is usually 1 on 1 but will vary with the type of material, from vertical in rock and 1 on 0.5 in stiff clay, to as much as, or more than, 1 on 2 in loose sand or fine sand with silt. The use of an earth auger on location surveys will enable the engineer to determine the most suitable side slope.

Ditches dug with dynamite usually contain much loose material. When this becomes stable, attention should be given to the shaping of bottom and side slopes so the ditch will be of the proper cross-section.

Grade.—Grades will vary from a minimum of 0.05% to a maximum of 1.5%, but the latter is not permissible except in stiff non-erosive soils. Long straight grades are preferable and abrupt changes should be made with a vertical curve. If possible, any variation in grade should have the increase down stream so as to prevent any loss of velocity and subsequent deposit of suspended material in the ditch. The depth depends upon the course the ditch must follow. The grade should begin at least 6 in. below the lowest point in the area to be drained, if sufficient fall is available.

Hand-dug ditches deeper than 10 ft are seldom satisfactory from a construction standpoint. Deep ditches should be dug with machinery so that proper disposition may be made of the excavated material. The bottoms should be finished by hand labor.

Spoil Banks.—The excavated material on small ditches should be scattered evenly on each side. If the ditch overflows, the dirt must be scattered, or, in large ditches, placed on the lower side. Sometimes it is impracticable to eliminate spoil banks entirely, particularly on large ditches. Frequent "eyes" should then be left, and if the flow through them into the ditch is considerable, the entrance should be at an angle, as in a branch or lateral.

The excavated material from seepage ditches must always be placed on the lower side: (a) To prevent its being washed back by surface flows; and (b) to prevent caving, which may occur when the upper marshy soil is loaded with the excavated material. A berm of at least 3 ft is left on all hand-dug ditches and not less than 8 ft to 10 ft for dragline ditches.

Stream Cut-Offs.—Where the new ditch is dug across the bends of an old stream, as is often done in rechanneling (Fig. 7), the excavated material can properly be utilized to fill the old run. The filling operation should always begin at the up-stream end of the cut-off at which point it is well to build a dam, consisting of logs, stumps, rip-rap, or other available material, in order



FIG. 7.—RE-CHANNELING CROOKED STREAMS

to force the stream through the new cut-off and avoid washing out the material used in filling the old stream bed. It is necessary to fill the pot-holes and bed of the old stream slightly higher than the grade of the new channels, the object being to have all water drain out promptly and permit no slack water after the stream has receded to normal flow conditions.

Seepage Ditches.—In draining a swamp the engineer must first determine the source of the water creating the swamp, whether from seepage outcrops, springs, or storm water retained in a natural basin.

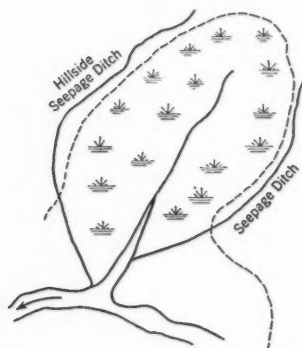


FIG. 8.—HILLSIDE SEEPAGE DITCHES

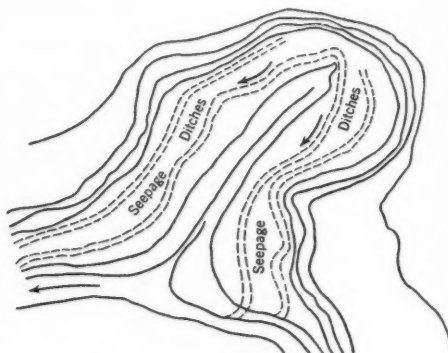


FIG. 9.—MULTIPLE SEEPAGE DITCHES

Seepage outcrops are caused by a change in permeability of the soil, such as sandy loam underlain by a clay subsoil. The flow of water down through the loam, arrested by the clay subsoil, flows along the top of this more impervious strata until there is an outcrop on a hillside, stream bank, or similar location. In draining such an area, narrow deep ditches are dug along the hillside just above the outcrop and into the impervious subsoil, intercepting the underground flow (Fig. 8). This ditch follows around the outcropping area

until all of the seepage has been intercepted, and is then continued to the stream or regular water course.

Sometimes a single line of deep-seepage ditches will not suffice and a series of parallel ditches will serve better (Fig. 9). This condition is created when the subsoil penetrated by the first ditch is sufficiently porous to permit water to seep from the ditch itself into the area below, and a new ditch is needed to arrest this flow.

These ditches should be only one spade wide on the bottom, and no rule can be laid down for depth except that they intercept the seepage outcrop. A 2-in. earth auger may be used in locating the depth of the water table and type of underlying subsoil, this information being indispensable in locating the ditch and establishing the grade.

If the seepage area is fed by springs (Fig. 10), it will be necessary to construct separate laterals to each spring. There will be occasions where a combination of methods must be used.

In deep seepage ditches where the flow of water is small, it is practical to build a culvert with medium-sized saplings, cut and placed in the bottom of the ditch, the limbs extending up stream. Some judgment will have to be exercised in determining the number of saplings to be used, keeping in mind that after the trench has been back-filled with the excavated material the voids left between the saplings must be large enough to drain all of the normal flow. Although sapling culverts will last for a long time in wet ground, the drainage problem is solved most permanently by open-joint drain tile laid in the seepage ditch.

Lateral Ditches.—Lateral or branch ditches should enter the main ditch at an acute angle on the same grade so that there will be as little disturbance of flow in the main channel as possible. In most cases lateral ditches need only be one spade wide at the bottom when they are used to carry residual water to the main drainage channel.

DITCH LINING

Progress in ditch lining in the United States has been slow. More research must be done in order to eliminate the tremendous maintenance costs of earth-lined drainage and irrigation ditches. This work, although more costly at first, eliminates the formation of pot-holes, prevents the destruction of property both public and private, reduces maintenance costs to a minimum, and obviates periodic oiling.

Tile.—For smaller ditches, or streams with a limited flow, tile is very satisfactory as a lining. Every tile project, no matter how small, should have

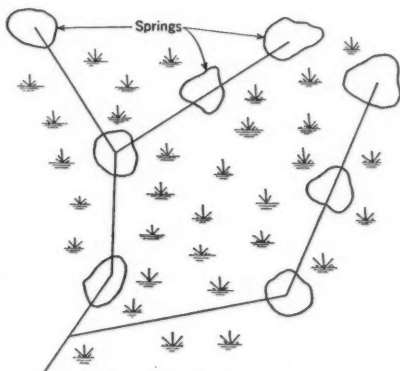


FIG. 10.—DRAINING SPRING-FED MARSHES

careful engineering supervision or it may defeat its purpose. A line with sagging joints may become a most obnoxious mosquito incubator.

Bell-and-spigot terra-cotta or concrete pipe is usually used for all conduits not involving seepage. Unglazed terra-cotta or concrete drain tile in 1-ft lengths is usually used in seepage areas. The same general instructions for laying will apply to both except that the joints are left open in seepage areas, where the tile must collect as well as carry off the water.

The trench is first constructed to approximate grade. At 25-ft intervals, and a constant height above grade, batten boards are set across the trench, a nail in each marking the center line. A string is then stretched over the batten boards between nails. The trench may then be graded accurately, checking the distance below the line with a template. The flow line of the tile is checked in the same manner.

Bell-and-spigot pipe is laid with the bells up stream. Beginning at the outlet end, each joint of tile must be laid on line, firmly embedded in the bottom of the trench and set accurately to grade.

The joints of conduit pipe are sealed with 1 : 1.5 cement-sand mortar to which has been added 15% of hydrated lime. In the smaller lines a "dead man" (a cement sack partly filled with sand) pulled through the finished line will insure the removal of rough mortar edges from the inside of the pipe. The "dead man" should be kept one joint back from the tile being laid.

Ditches must be back-filled carefully, the earth being tamped around the tile and some distance above, filling the trench well above the surface. In back-filling open-jointed tile it may be sufficient first to cover the joints with tar paper, or water-proof building paper, and then fill in with the excavated material; but better drainage can be obtained by covering first with 12 in. to 16 in. of washed gravel, topped off with fine sand.

Concrete.—Some large ditches through congested areas in cities have been lined with concrete. With no view to malaria control, many of these were built with wide, flat bottoms and straight sides. During periods of low flow, pools are formed which quickly become the breeding place for malaria-carrying mosquitoes.

Concrete-lined ditches and channels should be designed with the same care as closed sewers. The invert should provide for the minimum flow of water so as to prevent the deposit of suspended material.

For many years, engineers in the tropics have been engaged in paving the inverts of drainage ditches. Precast inverts are often used and their use should be encouraged here.

Officials of one progressive city have lined more than 10 miles of ditches in recent years. The bottom of the ditch is lined with concrete, and the sides are faced with rip-rap. Above the permanent lining the banks are sodded with Bermuda grass sod. It is very important that the banks be sodded when the concrete and rip-rap are laid, as this prevents water from seeping under and destroying the permanent lining. Sod laid in variable weather, and on the sides of ditches subject to flooding, should be pinned down. Long thin pieces of sod, properly pinned parallel to the ditch, are more economical than square pieces of sod. Curtain walls 2 ft deep should be constructed across the ditch

under the permanent lining to prevent undermining and destruction. The economical lining of drainage and irrigation ditches presents an interesting research field for the Engineering Profession.

CULVERTS

There are four important factors in the proper design of a culvert: (a) Size; (b) length; (c) cross-section; and (d) grade. The size is dependent on the area of water-shed, type of soil and topography; and the length is governed by the depth of fill and width of roadway over the culvert. The floor or invert of all culverts should have a circular or parabolic cross-section to provide sufficient velocity during dry-weather flow to prevent mosquito breeding.

Location engineers on highways and railroads should never set culvert grades until differential levels are run to determine if the low areas (swamps and ponds) on the upper side of the roadway can be drained. Failure to follow this procedure frequently makes it necessary, after construction, to cut the pavement in order to relay the culvert or lay a new one, to jack a culvert through the road-bed, or to cut out the bottom of an existing concrete culvert. These costly revisions can be avoided if careful consideration is given to establishing a correct grade when the culvert is first installed.

FILLING

Filling eliminates mosquito-breeding areas, and low spots should be filled whenever possible, as this method does not require continuous maintenance. Ponds may be filled with trash, old automobile bodies, ashes, and other débris, but must finally be covered with a top layer of earth. Any putrescible material used must be covered daily with street sweepings or earth to a depth of 1 ft. If the filling process is to continue over a long period, no putrescible material must be allowed in the fill as the nuisance created will be too great.

Permanent breeding areas in many towns and cities can be filled in this manner at almost no cost. In fact, in many cases, the average length of haul is shortened, and the filling results in a saving to the city. Property owners readily agree to this method, as an unsightly mosquito-breeding area can often be converted into potential revenue property at little cost.

The filling of pot-holes in marshy areas may eliminate the need of some lateral ditches and a saving in yardage in the main ditch. Pot-holes, either at the ends of culverts or in the bottom of ditches, are prolific sources of mosquitoes and may be difficult or impractical to drain. They should be eliminated by filling with brickbats, broken concrete or native stone, and, where practicable, concrete aprons or spillways should be constructed over these fills. This spillway, however, must be built so that further pot-holes will not be created beyond the fill.

PONDS

Stock ponds are essential in many sections of the country and cannot be eliminated. A dug well constructed in the center, or at one side and connected to the pond with a ditch, will retain water after the pond dries up. The pond should be stocked with *Gambusia affinis* (top-minnows). The same practice

may be used in undrainable borrow-pits or deep pot-holes in swampy areas. All ponds should have a definite slope to the lowest point so as to avoid shallow edges in which mosquito-breeding hoof-prints may be created.

DITCH CONSTRUCTION

Prior to actual construction, it is essential that all easements or rights of way be secured. Personal and property-damage waivers should be obtained from owners of property through which the ditch will pass. Lawsuits may be avoided in this manner.

Construction work is grouped into four phases: (a) Clearing; (b) grubbing roots and stumps; (c) roughing out the section; and (d) finishing the ditch. Large crews should be divided into four groups, to each of which will be assigned a definite phase. They will thus become proficient in their duties. The most accurate and reliable men should be put in the finishing crew. In small crews the first two phases may be done by the entire crew, returning to the ditching work later, but the best man, or men, must always be assigned to the finishing work.

Clearing.—In heavy ditching where machinery is used there is need of considerable room for operations. Ditching by hand labor does not require the same amount of clearing, and often removal of underbrush is all that is necessary.

Maintenance must be in the mind of every malaria control engineer from beginning to end. A ditch dug into the subsoil does not require as much maintenance work as shallow ditches, not only because the soil is more stable, but also because vegetation is retarded until favorable growing conditions are established. Shade is also a retarding factor for vegetation and algae. Too much clearing for ditch rights of way must be avoided.

Grubbing.—Grubbing of roots and stumps in the ditch cross-section should be thorough. Location lines should avoid large trees whenever possible, but when they do come within the ditch cross-section they should be removed completely, to a depth below the grade line, so as to avoid leaving exposed root fragments which will collect debris and form a dam in the stream.

Stump holes, either hand-dug or dynamited, should be back-filled by the finishing crew, when they extend into the bottom or into the ditch bank, so as to obtain a true grade and regular ditch cross-section.

Roughing.—Construction should begin at the outlet end and proceed upstream. It is assumed that the engineer has reset all center stakes damaged by clearing and grubbing operations and has marked each with the cut at that station. The width of the ditch is then roughed out with vertical sides to the approximate depth. Usually, this is done by first staking off half the bottom width on each side of the center stake, stretching a string, and marking on the ground with a square-pointed shovel. This bottom is always marked off at least 200 ft in advance of the construction crew.

Finishing.—The finishing crew completes the side slope and shapes the bottom. In finishing the center of the V-bottom, a round-point shovel is used for joining the bottom slopes. This is shown by a dotted line in Fig. 6.

A simple, cheap, and accurate method of maintaining a check on grades at all times has been described, as follows:

Before the roughing work begins the foreman drives poles at what will be the top of slope on each side of the center stake. He then measures the vertical distance from the ground to his eye. Suppose this is 5.1 ft; by leveling from the center stake, he marks a point on each pole which will be 5.1 above the grade line of the ditch. If the cut is 5.1 ft at that point, the pole will be marked at the ground surface; if the cut is 3.1 ft, the mark will be 2 ft above the ground surface, assuming that the ground is level. Strings fastened at these points and stretched across at each station will all be at the same height (5.1 ft), above the grade line of the proposed ditch. Standing erect at any point in the ditch the foreman can readily check the ditch grade by observing two or more strings.

The roughing and finishing crew will also find the strings helpful. The final grade should be checked by using a rod, and it is good practice for the finishing crew to use a template at intervals of 25 ft for checking the side slope.

Bridges should be constructed for all vehicular or regular cattle crossings. Foot logs, nailed to stakes to prevent removal, should be provided for pedestrian crossings. These precautions will obviate the necessity for unapproved crossings which so often result in ditch obstruction.

BARRICADES

The engineer should provide, erect, maintain, and, upon completion of the work, remove, all necessary barricades; provide suitable and sufficient red lights, danger signals and signs; provide a sufficient number of watchmen; and take all necessary precautions for the protection of the work and the safety of the public.

Highways closed to traffic must be protected by effective painted barricades of a type approved by the engineer. Warning and detour signs are placed at all closures and intersections along the highways and detour routes, for which he is responsible. The temporary detour route should be clearly indicated throughout its entire length. All barricades and obstructions must be illuminated at night, and all lights kept burning from sunset to sunrise.

EXPLOSIVES

Explosives must be handled carefully. They should be stored in a dry, well-ventilated place, safe from fire or bullets, at a safe distance from roads and dwellings, and must be guarded at all times. Dynamite freezes at 40° F. Care must be taken to protect it from freezing, as it is difficult to shoot after it has been frozen.

SANITATION

Sanitary facilities should be provided for large crews working for several days along one section of a ditch. Approval of the type of facilities should be secured from the Department of Health in the State in which the work is being done.

Ordinarily a fly-tight pit latrine with a canvas, burlap, board or brush screen is satisfactory. The pit is dug 2 ft wide, 4 ft deep, and 2 ft long for each unit of 20 men in the crew. A fly-tight latrine box 16 in. high and 6 in. longer and wider than the pit is placed over the pit. The extra length and width will allow a bearing surface all around. Earth is then banked 2 in. high all around the box.

A seat hole for each 20-man unit must have a self-closing fly-tight lid. A V-shaped urinal trough, providing 2 ft of length for each 20 men, can be drained through a pipe into one corner of the latrine pit. The latrine box can be made movable so it can be taken from place to place as the work progresses.

When the latrine is no longer needed in a particular location, the box is removed and the original excavated material is used to cover the feces and urine. The earth should be well tamped and the pit completely filled and mounded.

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The original draft of this series of reports was prepared under the chairmanship of E. L. Filby, Assoc. M. Am. Soc. C. E., then Chief Engineer of the Florida State Board of Health.

Collaborating with the sub-committee in editing the report were Dr. L. L. Williams, Jr., Senior Surgeon, in charge of Malaria Investigations, U. S. Public Health Service; Dr. T. H. D. Griffiths, Senior Surgeon, Malaria Research Laboratory, National Institute of Health; Dr. Mark F. Boyd, Rockefeller Foundation, Secretary, National Malaria Committee; and Dr. W. V. King, Senior Entomologist, Bureau of Entomology, U. S. Department of Agriculture.

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Representing the National Malaria Committee on the Joint Committee were: L. M. Clarkson, Director, Division of Sanitary Engineering, Georgia State Department of Public Health; and, G. H. Hazlehurst, Chief Engineer, Alabama State Board of Health. Those who represent the Society are: Philip G. Hasell, Assoc. M. Am. Soc. C. E.; and Louva G. Lenert and Nelson H. Rector, Members, Am. Soc. C. E.

Respectfully submitted

LOUVA G. LENERT, *Chairman*
of the Joint Committee

APPENDIX I

BIBLIOGRAPHY

Introduction.—

- (1) **Hoffman, Frederick L.** A Plea for a National Committee on the Eradication of Malaria. *Southern Medical Journal*, Vol. 9, No. 5, May, 1916.
- (2) **Maxcy, Kenneth F.** Malaria Distribution in the South. *Public Health Repts., Reprint No. 920*, Vol. 39, No. 20, May, 1924, p. 16.
- (3) **Dauer, C. C. and Faust, E. C.** Malaria Mortality in the Southern United States for 1934, with Supplementary Data for Previous Years. *Southern Medical Journal*, Vol. 29, No. 7.
- (4) **Industrial Relations in Malaria.** Annual Reports of Sub-Committee of National Malaria Committee. *Southern Medical Journal*, Vol. 29, No. 8, 1936.

Anopheles Mosquito and Malaria Transmission.—

- (5) **Carter, Henry R.** Malaria, Lessons on Its Cause and Prevention. U. S. Public Health Service, *Reprint No. 18*, Government Printing Office, Washington, D. C., 1915.
- (6) **James, S. P.** Malaria at Home and Abroad. London, XI, 1920.
- (7) **Freeborn, S. B.** The Mosquitoes of California. Univ. of California Publications, *Technical Bulletins*, Vol. 3, No. 5, 1926, pp. 333-460.
- (8) **Dyar, H. G.** The Mosquitoes of the Americas. Carnegie Inst., Washington, D. C., *Publication 387*, 1928.
- (9) **Matheson, R.** A Handbook of Mosquitoes of North America, Springfield, Ill., XVIII, 1929.
- (10) **Boyd, M. F.** An Introduction to Malariology. Cambridge, Mass., XIV, 1930.
- (11) **Headlee, T. J.** The Development of Mechanical Equipment for Sampling the Mosquito Fauna and Some Results of Its Use. *Proceedings, 19th Annual Meeting, New Jersey Mosquito Extermination Association.* New Brunswick, N. J., 1932, pp. 106-128.
- (12) **Curry, D. P.** Breeding of Anopheline Mosquitoes Among Aquatic Vegetation of Gatun Lake, Accompanied by Long Flights of *A. albimanus*, Wied. Reprint, *Southern Medical Journal*, Vol. 27, No. 7, 1934, pp. 644-651.
- (13) **Bradley, G. H. and McNeel, T. E.** Mosquito Collections in Florida with the New Jersey Light Trap. *Journal Economic Entomology*, Vol. 28, 1935, pp. 780-786.
- (14) **Komp, W. H. W.** A Guide to the Identification of the Common Mosquitoes of the Southern United States. U. S. Public Health Service, *Public Health Repts., Reprint No. 836* (revised 1936).
- (15) **Johnson, H. A.** Notes on the Occurrence of *A. walkeri*. *Southern Medical Journal*, Vol. 29, No. 8, 1936.
- (16) **Bradley, G. H.** On the Occurrence of *Anopheles walkeri*, Theobald, in Florida. *Southern Medical Journal*, Vol. 29, No. 8, 1936.
- (17) **Bradley, G. H.** On the Identification of the Mosquito Larvae of the Genus *Anopheles* Occurring in the United States. *Southern Medical Journal*, Vol. 29, No. 8, 1936.

Malaria and Impounded Water.—

- (18) **Carter, Henry R.** Impounded Water. Some General Considerations on Its Effect on the Prevalence of Malaria, *U. S. Public Health Service Reprint No. 244*, Government Printing Office, Washington, D. C.
- (19) —. Impounded Waters, Their Effect on the Prevalence of Malaria. Survey at Blewett's Falls. *U. S. Public Health Service Reprint No. 248*, Government Printing Office, Washington, D. C., 1915.
- (20) **LePrince, J. A.** Impounded Water on Coosa River, Alabama. *Public Health Repts.*, Reprint No. 257, 1915.
- (21) **Gage, Elliot H.** Impounded Waters and Malaria. *Public Health Repts.*, Reprint No. 1022, Vol. 40, No. 26, 1925. (In North Carolina.)
- (22) **LePrince, J. A.** Reservoirs and Malaria. *Public Works*, February-March, 1926.
- (23) —. The Relation of Storage Water Supply Lakes to Malaria, *Journal of the American Water Works Association*, January, 1926.
- (24) **Regulations Governing Impounding of Waters.** State Departments of Health, Alabama, Georgia, Tennessee, and Virginia.
- (25) **LePrince, J. A.** Mosquito Control in Relation to Impounded Water Supply. *Journal of the American Water Works Association*, January, 1927.
- (26) **Constable, E. W.** Malaria Control on Water Supply Reservoirs. *Journal of the American Water Works Association*, March, 1930.
- (27) **Clarkson, L. M.** Oil Mixture and Oil Equipment for Anopheline Larvae Control on Impounded Projects. *Southern Medical Journal*, April, 1929.
- (28) **Gill, D. G., Kiker, C. C., and Sims, L. C.** Practical Application of Paris Green as a Larvicide for Anopheles Control on Impounded Lakes in Alabama. *Southern Medical Journal*, April, 1929.
- (29) **LePrince, J. A. and Johnson, H. A.** The Development of a Power Dusting Device for Applying Paris Green as an Anopheline Larvicide. *Public Health Repts.*, Reprint No. 1280, 1929.
- (30) **Curry, D. P.** Mosquito Breeding Areas in the Vicinity of Panama City and Their Control by Dusting with Paris Green from Airplanes. *Annual Report of Health Department of the Panama Canal*, 1931, pp. 59-64.
- (31) —. Breeding of Anopheline Mosquitoes Among Aquatic Vegetation of Gatun Lake, Accompanied by Periodic Long Flights of *A. albimanus*, Wied. Reprint, *Southern Medical Journal*, Vol. 27, No. 7, 1934, pp. 644-651.
- (32) **Stromquist, W. G.** Malaria Control in the Tennessee Valley. *Civil Engineering*, Vol. 5, No. 12, 1935, pp. 771-774.
- (33) **Watson, R. B.** Some Preliminary Observations on Airplane Dusting for *Anopheles* Larvae Control. *Southern Medical Journal*, Vol. 29, No. 8, 1936.
- (34) **Kiker, C. C.** Observations on Mosquito Control on Lake Wilson. *Southern Medical Journal*, Vol. 29, No. 8, 1936.
- (35) **Bishop, E. L.** Malaria Control Activities of the Tennessee Valley Authority. *Public Health Repts.*, Vol. 51, No. 29, 1936, pp. 970-975.

Malaria and Transportation.—

- (36) **Derivaux, Robert C.** The Relations of the Railroads in the South to the Problem of Malaria and Its Control. *Public Health Repts., Reprint No. 480*, Vol. 33, No. 31, Government Printing Office, Washington, D. C., 1918.
- (37) **Saville, Chas. and Van Hovenberg, H. W.** Malaria Control Engineering in Texas Increases Labor Efficiency. *Engineering News-Record*, 1918.
- (38) **St. Louis Southwestern Ry. Lines.** Malaria Control, a Demonstration of Its Value to Railroads Based on the Experience of the St. Louis Southwestern Railway Lines, 1917-1920.
- (39) —. Malaria and the Engineer. Prepared by Committee on Sanitary Engineering, National Malaria Committee, 1922.
- (40) **Fuchs, A. W.** Railroad Malaria Surveys. Missouri Pacific R.R., U. S. Public Health Service, *Bulletin No. 135*, 1922.
- (41) **Griffitts and Griffitts:** Mosquitoes Transported by Airplanes. *Public Health Repts.*, Vol. 46, 1931, pp. 2775-2782.
- (42) **Griffitts, T. H. D.** Air Traffic in Relation to Public Health. *American Journal of Tropical Medicine*, Vol. 13, 1933, pp. 283-290.

The Control of Malaria.—

- (43) **LePrince, J. A.** Control of Malaria. Oiling as an Anti-Mosquito Measure. *Public Health Repts., Reprint No. 260*, 1915.
- (44) —. Malaria Control: Drainage as an Anti-Malaria Measure. *Public Health Repts., Reprint No. 258*, 1915.
- (45) **Barber, M. A. and Hayne, T. B.** Arsenic as a Larvicide for Anopheline Larvae. *Public Health Repts.*, Vol. 36, 1921, pp. 3027-3034.
- (46) **Rose, Wickliffe.** Field Experiments in Malaria Control. International Health Board, 1919.
- (47) **Hardenburg, W. E.** Mosquito Eradication. McGraw-Hill Book Co., Inc., New York, N. Y.
- (48) **Covell, Gordon.** Mosquito Control (By Anti-Mosquito Measures). W. Tucker Co., London, England.
- (49) **LePrince, J. A.** Historical Review of Development of Control of Disease-Bearing Mosquitoes. *Transactions*, Vol. 92 (1928), pp. 1259, 1322.
- (50) **U. S. Public Health Service.** *Transactions of the First-Fifth Conference of Malaria Field Workers.* *Public Health Bulletins*, Nos. 104, 115, 125, 137, and 156. Government Printing Office, Washington, D. C., 1919-1924.
- (51) **Metz, Charles W.** Malaria Control Through Mosquito Eradication. *Public Health Repts., Reprint No. 500*, Vol. 34, No. 5, 1919, pp. 167-183.
- (52) **Vannate, Robert, L. and Ginsburg, Joseph M.** Practical Application of Pyrethrum Larvicide.
- (53) **Rector, Nelson H.** The Execution of Minnow Drainage. Mississippi State Board of Health, *Bulletin No. 1*.
- (54) **National Malaria Committee.** Improved Housing as a Factor in Malaria Control. Symposium on Malaria. *Southern Medical Journal*, 1932.
- (55) **Coogle, C. P.** The Screening and Mosquito Proofing of Tenant Houses. *Alabama State Board of Health Bulletin*.

- (56) **Hinman, E. Harold.** Recent Developments in Methods of Mosquito Control in Anti-Malaria Campaigns: Symposium on Malaria. *Southern Medical Journal*, 1937.

General.—

- (57) **Elliot, Chas. G.** Engineering for Land Drainage. John Wiley & Sons, Inc., 1919.
- (58) **Powers, W. L. and Teeter, T. A. H.** Land Drainage. John Wiley & Sons, Inc., 1922.
- (59) **Parson, J. L.** Land Drainage. Myron C. Clark Pub. Co., 1915.
- (60) **Pickels, George W.** Drainage and Flood Control Engineering. McGraw-Hill Book Co., Inc., New York, N. Y., 1925.
- (61) **Griffitts, T. H. D.** Malaria and Anti-Malaria Drainage. Florida Health Notes, Vol. 26, No. 2, February, 1934.
- (62) **White, C. M.** Design and Specifications of a Drainage System. North Carolina State Board of Health Publication.
- (63) **Precast Concrete Sectional Inverts for Paving Drainage Ditches.** Health Department, The Panama Canal, C. Z., Reprint.
- (64) **Fletcher, A. H.** A Permanent Type of Ditch Construction. *American Journal of Public Health*, Vol. 25, No. 8, August, 1935.
- (65) **Magoon, E. H.** Forms for Precast Concrete Inverts. *Boletín de la Oficina Sanitaria Panamericana*, October, 1936.
- (66) **Scharff, J. W.** On a Method of Planting Grass in Connection with Anti-Malarial Projects. *Malayan Medical Journal*, December, 1935. Abstracted *Tropical Diseases Bulletin*, Vol. 33, No. 10, October, 1936, p. 771.
- (67) **Mosquito Control Engineering.** *Engineering News-Record*, Vol. 117, 1936.
- I. The Growth and Importance of Anti-Mosquito Work, by L. O. Howard, July 23, 1936, pp. 118-122.
- II. Mosquitoes: Species and Habits, by Dr. T. J. Headlee, August 6, 1936, pp. 199-201.
- III. Control Technique and Organization, by Russell W. Gies, August 13, 1936, pp. 225-228.
- IV. Inland Control Methods, by J. Lyell Clarke, August 20, 1936, pp. 266-269.
- V. The Salt Marsh Problem, by R. J. Van Derwerker, August 27, 1936, pp. 304-307.
- VI. Effective Malaria Control, by Dr. L. L. Williams, Jr., September 3, 1936, pp. 341-343.
- VII. Ending Malaria in New Mexico, by C. M. Adams, September 10, 1936, pp. 372-374.
- VIII. Mosquito Abatement in Delaware, by W. S. Corkran, September 10, 1936, pp. 374-376.
- IX. Malaria and the Mississippi Valley, by J. A. LePrince, September 17, 1936, pp. 404-405.
- X. California's Campaign, by Harold F. Gray, September 17, 1936, pp. 405-406.

APPENDIX II

FORMS

The following forms illustrate the permits and inspection record as used in one State governing the impounding of water.

STATE BOARD OF HEALTH
PRELIMINARY PERMIT

For the Impounding of Waters
Issued to

Name:

Post Office:

County:

Application for a permit, giving necessary data as specified in sections one and two of the "Regulations Governing the Impounding of Waters," having been made, a Preliminary Permit to proceed with the construction of this project is hereby issued,

this day of 19

Final permit will be issued when it has been demonstrated that the health of those affected by the project has been properly safeguarded. The provisions of the "Regulations Governing the Impounding of Waters," a copy of which is attached and made a part hereof, shall apply except as may be hereinafter exempted in writing.

.....
State Health Officer

STATE BOARD OF HEALTH
PERMIT

For the Maintenance of an Impounded Water Project
Issued to

.....
The construction of an impounded water project, described in an application for a Preliminary Permit, which Preliminary Permit was granted on the day of, 193..., having been completed in accordance with the terms of the said Preliminary Permit, this Permit to Maintain the Impounded Water Project is issued this day of, 193....

The "Regulations Governing the Impounding of Waters," a copy of which Regulations is attached and forms a part of this Permit, shall be observed except as exempted in writing. This Permit shall remain in force so long as the holders thereof maintain said reservoir in a condition which does not render it a menace to the public health.

.....
State Health Officer

MALARIA
IMPOUNDED WATER INSPECTION RECORD

STATE BOARD OF HEALTH

Date.....County.....Name of Pond.....Size in acres.....
Owner.....Owner's Address.....
Date Impounded.....
Location of Pond.....Form Letter Sent.....
Application received.....Preliminary permit issued.....
Authority to impound granted.....Maintenance permit issued.....
Following inspections made.....193.....Pond impounded.....
Basin cleared.....Pond stocked with *Gambusia*.....Size drain in
base of dam.....
Basin cleared of original growth.....Amount of aquatic
growth.....
Source of water supply.....Approximate g.p.m. inflow.....
Control Measures: Water level fluctuation.....
Gambusia.....
Cleaning shore line.....Oiling.....Paris green.....
Number of dips made.....Number of *Anopheles* larvae found.....
Number of *Anopheles* larvae identified as *A. quadrimaculatus*.....
Number of continuously inhabited houses, barns, or chicken houses within
 $\frac{1}{2}$ mile of pond searched for adult *A. quadrimaculatus* mosquitoes.....
Total number of adult *A. quadrimaculatus* mosquitoes found.....
Describe any other typical *Anopheles* breeding area in the vicinity of the
impounded pond.....
.....
Number of people living within 1 mile of the pond.....
Malaria reported within 1 mile of the pond.....
Conference with the owner of the pond.....
Recommendations and Remarks.....
.....
.....
.....
.....

Signature of Person Making Inspection

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

GRIT CHAMBER MODEL TESTS FOR DETROIT, MICHIGAN, SEWAGE TREATMENT PROJECT

Discussion

BY GEORGE E. HUBBELL, ASSOC. M. AM. SOC. C. E.

GEORGE E. HUBBELL,²⁰ ASSOC. M. AM. SOC. C. E. (by letter).^{20a}—At this time the writer has little further to offer on the Detroit grit chamber model tests. Only tests of the finished structure itself will suitably close this discussion. However, there appears to have been some misunderstanding of the writer's conclusions. Professor Ridenour's interpretation of the data leads him to believe that shallow grit chambers are more efficient than deep ones. In Table 6, the comparison of removal on the basis of percentage of total sand is questionable since the actual pounds of the various sizes used in the two tests differ widely. Furthermore, it would appear that the critical size at which the $\frac{1}{V_h}$ -ratio equals the $\frac{d}{V_s}$ -ratio is best indicated by the size of sand that is completely removed. For the Grand Rapids test, this is 0.295 mm to 0.417 mm; and for Dearborn it is 0.2 mm, the Dearborn $\frac{L}{d}$ -ratio equaling 29.8. The equivalent length for Grand Rapids, therefore, would be 179 ft, and the tests indicate a critical size of 0.2 mm, as would be expected if the relationship $\frac{1}{V_n} = \frac{d}{V_s}$ is substantially correct. With regard to the average size of sand removed in the equivalent section of the two chambers, the different character of the sands involved should be considered, the Grand Rapids test sand having an over-all average size of 0.57 mm as compared to 0.42 mm for the Dearborn sand. The writer does not conclude that slightly greater removal efficiency can be expected from the deeper tank, but rather that the same degree of efficiency can be obtained with deep chambers properly proportioned.

Professor Camp has developed, in a masterly manner, the theoretical grit-

NOTE.—The paper by George E. Hubbell, Assoc. M. Am. Soc. C. E., was published in December, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1938, by G. M. Ridenour, Assoc. M. Am. Soc. C. E.; April, 1938, by Thomas R. Camp, M. Am. Soc. C. E.; and September, 1938, by W. E. Howland, Assoc. M. Am. Soc. C. E.

²⁰ Cons. Engr. (Hubbell, Roth & Clark, Inc.), Detroit, Mich.

^{20a} Received by the Secretary December 27, 1938.

chamber efficiency to be expected, based on the laws governing sedimentation in quiescent water, and has shown that the efficiencies actually obtained in the Grand Rapids, Dearborn, and model tests, and estimated for the Detroit grit chamber, all in flowing water, are about 10% less than those theoretically expected. These results appear to indicate a reasonably approximate measure of the effects of cross currents and unequal distribution of velocities which prevail in grit chambers under the usual operating conditions. It was assumed arbitrarily that a prototype velocity of 5 ft per sec would remove scum and grease, and no conclusion was reached from the model test other than that, if 1.29 ft per sec were adequate in the model, the higher velocity would be adequate in the prototype.

Professor Howland has raised some interesting questions with regard to the relationship between the transporting power of flowing water and grit-chamber design. Considerable information on the transporting power of streams is available, but it is difficult to apply it directly to grit-chamber design. The average velocity in the model was 0.26 ft per sec, and the sand used had a diameter of approximately 0.1 mm. Competent velocity computations would tend to indicate that sand of this size is critical and would be transported. Referring to the prototype, it would appear from similar computations that a sand, for example, of 0.35 mm would not settle at a velocity of 1.0 ft per sec since the competent velocity for such a particle size is variously estimated at from 0.5 to 0.7 ft per sec. Furthermore, the writer believes that any general expression that does not take cognizance of particle size is, of necessity, limited in usefulness, because there can be no question that, with reference to sedimentation, settling velocities, particle sizes, and tank velocities are intimately related. In so far as the writer knows, no data are available on the effect of concentration on removals.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

RELATIVE FLEXURE FACTORS FOR ANALYZING CONTINUOUS STRUCTURES

Discussion

BY RALPH W. STEWART, M. AM. SOC. C. E.

RALPH W. STEWART,³⁹ M. AM. SOC. C. E. (by letter).^{39a}—In presenting a new approach to the analysis of continuous structures it cannot be expected that the profession will adopt it quickly. The writer feels gratified, therefore, that of those contributing discussion several engineers have endorsed the method for the class of structures treated.

Mr. Shapiro thinks that the unpublished method of "transmission coefficients"⁹ is the best for all-round use. By private correspondence with Mr. Shapiro the writer obtained a solution of Fig. 25,⁴⁰ by the method of transmission coefficients and found that the number of arithmetical operations necessary to solve it by that method was greater than by the method of relative flexure factors; and that the former method lacked the automatic checks and valuable by-products offered by the latter method.

The direct construction by Mr. Peterson of influence lines for both moments and shear, and also for yielding supports by using a traverse of the elastic curves in connection with the Müller-Breslau relationship between influence lines and elastic curves, is a valuable addition to the paper. Any computer who wishes to avoid the graphical construction of funicular polygons to obtain the ordinates of the final elastic curves can use the following formula which, for beams of constant section, expresses the ordinates to the elastic curves in terms of the end slopes:

$$\text{Ordinate} = (\theta_1 k_1 k_2^2 + \theta_2 k_1^2 k_2) l \dots \dots \dots (23)$$

NOTE.—The paper by Ralph W. Stewart, M. Am. Soc. C. E., was published in January, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1938, by Messrs. Frederick Shapiro, and Dean F. Peterson, Jr.; May, 1938, by Messrs. George W. Housner, Homer M. Hadley, Adolphus Mitchell, John B. Wilbur, and Leon Blog; and June, 1938, by Messrs. A. Floris, D. B. Hall, Ralph W. Hutchinson, Arthur B. McGee, and E. Neil W. Lane.

³⁹ Engr. of Bridge and Structural Design, City of Los Angeles, Los Angeles, Calif.

^{39a} Received by the Secretary December 13, 1938.

⁹ "Moving Loads on Beams with Restrained Ends," by R. G. Brumfield. (Complete manuscript filed for reference in Engineering Societies Library, 33 West 39th Street, New York, N. Y.)

⁴⁰ "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), Fig. 1, p. 5.

in which θ_1 and θ_2 are the left and right end slopes respectively, and k_1 and k_2 the fractions of the beam length from the left and right ends respectively.

The computation and tabulation of ordinates from this formula will be less laborious to most computers than the graphical construction.

Mr. Housner objects to considering the length of the base of a traverse triangle as equal to the lengths of the two inclined sides. If the validity of the

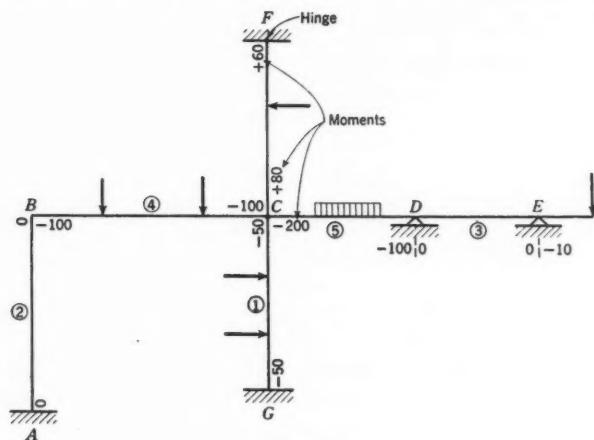


FIG. 25

traverse method of treating elastic structures is rejected on this account, it will be necessary also to reject the classic theory of flexure which neglects the difference between the abscissa of a point in a beam and the diagonal distance to the same point after the beam is deflected. Textbooks on flexure are based on the radius of curvature of a flexed beam being taken as

$$r = \frac{(dl)^3}{dx \, d^2y} \dots \dots \dots (24)$$

Treatises on the calculus show that the radius of curvature is

$$r = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} \dots \dots \dots (25)$$

An attempt to reduce Equation (25) to Equation (24) will disclose a point in the treatment of flexure used in all texts to which Mr. Housner's objections would be applicable. The value of an infinitesimal, or nearly infinitesimal, angle may be taken as equal to the value of either its sine or its tangent. In stress computations for bridges and buildings it is entirely proper and customary to consider the angles due to elastic deformation as conforming to the law of infinitesimal quantities, as is done in this paper.

The illustration by Professor Wilbur that for a continuous beam the same set of arithmetical computations used in the writer's paper can be applied in

connection with the moment-area method is correct. However, the extension of the moment-area delineation to more complex problems will give rise to difficulties, whereas the traverse delineation will make it easy for the computer to keep account of what he is doing. Mr. Hadley's complimentary statements about the flexure-factor method are very pleasing. Mr. Blog gives interesting general comments and opinions on various methods of analyzing continuous structures. As time passes and new literature is published the rating of these various methods will become determined more accurately. Messrs. Mitchell, Floris, Hall, and McGee endorse the flexure-factor method for one-story structures, and all have done constructive work to expand the scope of the paper. The statement of Mr. Mitchell that for continuous beams the three-moment theorem and end-moment distribution cannot compare with the flexure-factor method, and the statement of Mr. McGee that he has adopted the method for rigid frames, are very gratifying.

The solution by Mr. McGee of a laterally loaded unsymmetrical frame with one partly fixed pier base supporting a tapering pier is an interesting illustration of the utility of the flexure-factor method. This solution is worth saving for reference, not only on account of the problem but also because of the complete exposition of the method of adopting the Ruppell tables³⁴ to an elastic curve traverse.

Mr. Hutchinson expresses an opinion that the relative flexure-factor method is advantageous only for continuous spans on "flexible piers," and that in cases in which the elastic properties of the piers must be considered, or in which a hinge occurs in a span, the method is less convenient than other methods. According to studies by the writer and others, the inclusion of elastic piers adds no more to the work of computation than if methods other than the flexure-factor method are used. If side-sway can be neglected it adds less work to include piers in a flexure-factor analysis than to include them in a moment distribution or slope deflection analysis. It is also easy to traverse across a hinged span as the following solution will demonstrate. Fig. 26 represents the

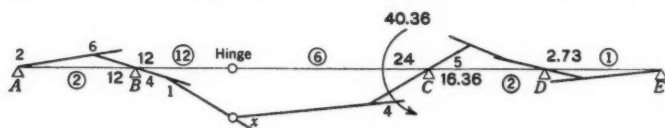


FIG. 26

same continuous beam as Fig. 2 except that a hinge has been introduced at the third-point of Span BC. The original $\frac{I}{l}$ -value for this span was 4. After inserting the hinge the $\frac{I}{l}$ -values of the two fractional parts will become 12 and 6, as shown.

By statics, the relationship of M_{BC} to M_{CB} must be in proportion to the distances of Points B and C from the hinge, or as 1 : 2. Dividing by the stiffnesses, the ratio of the corresponding Δ -angles is as 1 to 4. Write these

³⁴ *Transactions, Am. Soc. C. E.*, Vol. 90 (1927), pp. 167-187.

relative Δ -values on the figure; multiply them by the stiffnesses to get the relative moment values; designate the angle at the hinge by x ; balance the moment of 12 with M_{BA} also equal to 12; and write the angle values appurtenant to $M_{BA} = 12$, in Span AB . For convenience, consider Span BC as divided into nine equal units of length. The usual type of traverse equation then gives $(4 \times 9) + (1 \times 8) - 6x - (4 \times 2) = 0$; and $x = 6$. Combining angles from Points B to C gives $\theta_c = 5$.

The resisting moments in Spans CD and DE will now be $\frac{5}{11}$ times those in Fig. 2. Write these in the figure, add the two resisting moments at Point C to get the activating (fixed-end) moment of 40.36 at Point C , and the picture of the flexure is complete with all moments and rotations given. If the hinge is placed at one-seventh of the distance from Point B to Point C (which is the point of contraflexure in Fig. 2(e)) the angle x will be found by the foregoing procedure to be zero, and the hinge will be without effect on the moments in the structure. This form of analysis will enable the computer to determine the desirable location for a hinge more quickly than by other forms.

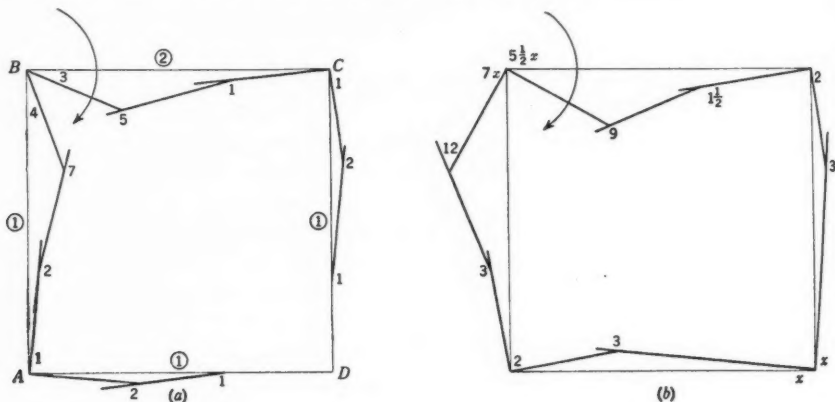


FIG. 27

Mr. Hall's solution of the closed box illustrates very well how an exceedingly accurate approximation of the bending moments can be obtained by a simplified application of the method. Although it is as exact as the common theory of flexure, the solution given in the paper involves the somewhat troublesome mental effort of carrying computations along the figure with each angle expressed as a two-term quantity. After further study the writer has concluded that most computers will find it easier to make two separate computations for this structure, using a single term for each angle. This is done as follows:

In Fig. 27(a), consider rotation as zero at Joint D and draw a traverse DCB due to the moment M_{BC} as shown, ending with a joint rotation of 3 at Point B . Now, with the moments about Joint D in balance, begin at D and draw the traverse DAB , as shown, ending with a joint rotation of 4 at Point B . To equalize these joint rotations at Point B it is obvious that Joint B , and

The remainder of the solution involves nothing that has not already been explained. Bending moments not necessary for the computation are omitted from Figs. 28(a) and 28(b), for simplicity. Any omitted moment may be added to the figure at once by multiplying its Δ -angle by the stiffness factor of the member.

Mr. Lane's solution of an unsymmetrical frame with a sloping column is of interest. It avoids the use of trigonometric functions which were applied to the solution of the same type of problem by Mr. Mitchell. His statement that the flexure-factor method is time saving is appreciated.

In conclusion it may be stated that the method of flexure factors based on a traverse of the elastic curves is not as restricted in its scope as some of the discussion indicated. It can be extended to curved beams and arch analysis. For arch analysis the Δ -angles are not in the unsprung axes of the curved members but are at the intersections of those end slope tangents which are selected to control the design. The ordinates of these Δ -angles can be computed, and by using the Δ -angle points as centers of rotation, the usual arch formula can be computed. Some of this extension work might be suitable for college theses.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

FLOOD ROUTING

Discussion

BY EDWARD J. RUTTER, AND QUINTIN B. GRAVES, ASSOC. MEMBERS,
AM. SOC. C. E., AND FRANKLIN F. SNYDER, JUN. AM. SOC. C. E.

EDWARD J. RUTTER,⁴⁵ AND QUINTIN B. GRAVES,⁴⁶ ASSOC. MEMBERS, AM. SOC. C. E., AND FRANKLIN F. SNYDER, ⁴⁷ JUN. AM. SOC. C. E. (by letter).^{47a}—The writers appreciate the interest shown by the discussions of this paper. Several interesting suggestions have been given which add to the treatment of the subject.

Mr. Camp raises the question as to the accuracy of discharges at a "routing station" which is affected by confluence with another stream. This factor is an important feature in any stage-discharge relation, and the more accurately the relation is known the more accurate will be the discharge and elevation obtained by the routing method described in the paper. It should be noted, in studies of the 1937 flood on the Lower Tennessee River, mentioned by Mr. Camp, that the reason observed stages at the Gilbertsville, Ky., dam site were not checked is that the rating curve had to be extended into a region where no measurements of flow were available and an extremely high submergence obtained.

Mr. Goodrich offers an improvement on his method of routing with a single discharge curve and single capacity curve. It does not seem possible, however, that the graphical-mechanical solution is more rapid than his earlier method, which was readily adapted to the writers' problem of using sloped storage instead of level storage.

Mr. Myers mentions several flood studies which have been made and compares them in a general way with those on the Tennessee River. As the flood wave travels down stream the effect of up-stream tributary reservoirs tends to

NOTE.—The paper by Edward J. Rutter, and Quintin B. Graves, Assoc. Members, Am. Soc. C. E., and Franklin F. Snyder, Jun. Am. Soc. C. E., was published in February, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1938, by Messrs. Cecil S. Camp, and R. D. Goodrich; September, 1938, by Messrs. E. L. Myers, and Ralph W. Powell; and October, 1938, by Messrs. William T. Collins, George E. Townsend, and Waldo E. Smith.

⁴⁵ Hydr. Engr., Tenn. Valley Authority, Knoxville, Tenn.

⁴⁶ Instr. in Civ. Eng., Dept. of Civ. Eng., Univ. of Texas, Austin, Tex.

⁴⁷ Hydr. Engr., Dept. of Forests and Waters, Commonwealth of Pennsylvania, Harrisburg, Pa.

^{47a} Received by the Secretary January 6, 1939.

become less, the percentage of uncontrolled local area becomes larger, and the natural hydrograph becomes broader. All these factors combined tend to reduce the reservoir storage effect, provided there is insufficient main-river reservoir storage to counterbalance that tendency.

Mr. Powell has demonstrated a procedure for routing floods in the "natural condition" through a reach of a river for which the storage quantities were based on a study of past floods and not necessarily on storage obtained from topographic maps or cross-sections. He has, effectively, eliminated the storage loop that is obtained when storage is plotted against outflow alone. This requires accurate stream-flow records to determine the values of storage in the reach for various values of inflow and outflow. Difficulty might be experienced if it were necessary to extend these data to take care of floods considerably larger than any experienced to date, although the curve of Fig. 11 could be extended reasonable amounts.

It is desirable to use the same routing method in the "natural condition" as in the condition with the dam in place. No accurate check can be made on any procedure for routing through a reservoir created by a dam until after the dam is built. The writers believe that by treating the two cases as nearly as possible in the same manner, and by making as accurate a check as possible on the "natural condition" routing, the results obtained for the case with the dam in place will be more reliable.

The writers have applied Mr. Powell's procedure to several different conditions and have found that it will give satisfactory results. It should be noted, in his discussion, that the inflows used in Fig. 11 are also used in the routing

TABLE 6.—COMPARISON OF OUTFLOW RATIO O_1 , APPLIED TO CHICKAMAUGA (TENN.) REACH

(a) 1936 FLOOD						(b) 1917 FLOOD					
Period	Writers' method	Powell's method	Period	Writers' method	Powell's method	Period	Writers' method	Powell's method	Period	Writers' method	Powell's method
(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
March:			April:			February:			March:		
16	25	25	1	117	118	18	43	43	6	310	310
17	29	29	2	97	93	19	55	54	7	332	336
18	36	36	3	136	129	20	85	82	8	330	335
19	46	48	4	167	173	21	120	120	9	278	285
20	55	54	5	156	159	22	142	144	10	180	177
21	62	65	6	140	137	23	140	141	11	103	98
22	77	76	7	175	173	24	127	127	12	87	89
23	74	74	8	217	220	25	101	100	13	86	85
24	71	72	9	217	222	26	83	88	14	96	96
25	93	91	10	191	190	27	83	81	15	118	118
26	143	141	11	149	148	28	80	80	16	118	117
27	185	187	12	113	113	March:			17	111	110
28	225	223	13	91	93	1	83	81	18	126	126
29	243	246	14	78	79	2	115	110	19	143	144
30	235	239	15	68	69	3	165	167	20	145	147
31	188	180	16	62	62	4	214	211	21	135	135
....	17	58	56	5	268	265	22	124	120
....	18	52	52

shown in Table 5, and that all results depend on the routing method of the writers and are not taken from any really natural flows. This does not constitute an adequate check on the method for use with other floods; hence, the

writers used Fig. 11 on flows for the Chickamauga reach for February and March, 1917, and for March and April, 1936, the results (Table 6) of which indicate very clearly that Mr. Powell's procedure will work satisfactorily. His procedure was then tried on three other reaches and the factor, $0.4 I + O + S$, was found to vary according to the conditions of the reach. Having found the curve for another reach (similar to Fig. 11), other floods were routed using this curve. All results indicate a workable procedure.

Another condition that affects the value of the coefficient 0.4 is the relation between the up-stream flood and the local flood. A comparison was made between Mr. Powell's method and the writers' method for two cases—(1) assuming no local flood; and (2) assuming no up-stream flood. As expected, the peak storage value was smaller in the first case and larger in the second as compared to that from the curves in the paper. Table 7 shows the comparison for the two cases.

TABLE 7.—RELATION BETWEEN UP-STREAM FLOW AND LOCAL FLOW

Day	(a) NO LOCAL FLOOD					(b) NO UP-STREAM FLOOD				
	I_1	Powell		Writers		I_1	Powell		Writers	
		O_1	S_1	O_1	S_1		O_1	S_1	O_1	S_1
December:										
21	45	37	48	37	48	45	37	48	37	48
22	92	60	88	60	88	45	45	56	45	56
23	110	99	131	100	130	53	47	62	49	60
24	153	124	171	124	169	68	59	77	61	71
25	201	165	236	160	239	82	72	96	76	84
26	202	195	279	194	288	80	82	104	81	89
27	213	202	297	200	309	76	78	100	78	86
28	222	212	318	211	333	79	77	100	77	86
29	201	216	313	216	329	74	76	100	77	85
30	191	204	285	205	300	62	72	88	67	77
31	157	187	242	187	256	50	58	70	56	66
1*	124	150	186	155	195	42	47	57	46	56

* January 1.

For the "natural condition," Mr. Powell's procedure is simpler than the writers' method, as far as curve reading is concerned, but required calculations are increased; hence, the type of procedure to use depends upon personal preference, or upon original information available.

Values of $0.4 I + O + S$ were computed for the Chickamauga reach for the case with the dam in place, with regulation to several different elevations and with no regulation. These were plotted against storage, and intermediate curves for actual use were interpolated. With this set of curves, direct routings are possible without resorting to a trial-and-error procedure as in the writers' method. However, additional arithmetical work is involved, as in the "natural condition." There is one other point to consider, namely, the $(0.4 I + O + S)$ -curves for use with the dam in place must be taken from routing curves similar to those developed by the writers when making a study of a dam which is to be constructed.

Mr. Collins' criticism of the assumed profile which gives a decrease in flow at the mouth of a large tributary at the beginning of a rise (December 22) is a

valid one. However, the actual value of storage cannot be greatly in error since the large tributary is near the middle of the reach. The assumed flows at the upper end of the reach, just above the tributary, just below the tributary, and at the lower end, are 76 000, 74 000, 62 000, and 60 000 cu ft per sec, respectively. The actual flows at the same points are approximately 76 000, 62 000, 74 000, and 60 000 cu ft per sec, respectively.

Mr. Collins is correct in his statement that, for some reaches, it may be better to treat tributary flows separately and add them to the routed main stream outflow at the lower end of the reach. This is particularly true of a tributary that enters the reach close to the lower end, and, consequently, has a relatively small effect on the storage values. However, by the same reasoning, the flow from a tributary which enters near the upper end of a reach should be routed with the main stream flow. Considerable study is necessary to determine if, and when, separate routing of tributary flows would be required. Each reach must be treated as a separate problem, and the procedure that gives the best results should be used. As stated in the paper (see heading, "Reaches"), the ideal procedure is to have routing stations at the mouth of all large tributaries.

Mr. Townsend presents a comparison of routing with 24-hr and 6-hr time periods, with the same inflow. In the case shown, the inflow has a relatively sharp peak, due to the method of operation used, and hence the reach assumes the characteristics of one in which a shorter time period than 24 hr gives more reasonable results. The method of checking the arithmetic described by Mr. Townsend is convenient and rapid. However, it does not check the reading of curves; these must be read again for a complete check. Mr. Townsend also notes several points in which improvements might be made in the procedure but gives no suggestions as to the method of making these improvements. Possibly more study would reveal better methods for the determination of the non-steady flow profiles in some reaches.

Mr. Smith is partly correct in his statement that, if the time of travel in a reach is 24 hr, 12-hr, 6-hr, or 4-hr intervals can be used as satisfactorily as 24-hr intervals, although this would not be the case for a reach with little local or tributary inflow. Since the interval should be determined by the stream characteristics, the use of shorter intervals in routing adds to the work without materially increasing the accuracy. The same base data used in plotting the outflow-plus-storage curves for 24-hr periods can be used to plot similar curves for any time interval desired. For example, if a 6-hr routing period is required, the outflow O would be plotted against $O + 4S$, instead of $O + S$.

The back-water profiles shown in Figs. 3 and 4 have the same general appearance as those in Fig. 2 for the natural condition. At the time these curves were computed, every reasonable attempt was made to eliminate any sharp changes of slope, and the lines shown represent, as nearly as possible, the profiles that might occur. Probably, after the reservoir has been constructed and operated through several floods, a revision could be made in the shape of the profiles of water surface.

Mr. Smith questions the speeding up of flow through the reservoir. The writers have based the storage on the back-water profiles. If the profiles are

correct for the flow conditions indicated, the discharge at the dam cannot increase faster than the corresponding inflows, profiles, and storage allow. The change in time of travel is shown in the shape of the profile, and as a result, the value of storage is different.

The value of O_1 , in the first instance, need not be assumed nor estimated for an actual flood if there is a gaging station at the routing station. For hypothetical floods and at all routing stations where there has been no gaging station the initial value of O_1 must be estimated.

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DISCUSSIONS

PIN-CONNECTED PLATE LINKS

Discussion

BY BRUCE G. JOHNSTON, ASSOC. M. AM. SOC. C. E.

BRUCE G. JOHNSTON,¹¹ ASSOC. M. AM. SOC. C. E. (by letter).^{11a}—The discussion presented by Mr. Hussey is entirely constructive in nature. The close agreement between the tests of two eye-bars and the strength and type of failure predicted by Equations (2), (3), and (4) is very gratifying. It is unfortunate that these are the only two tests of eye-bars brought to light in discussion. Equations (63a) and (63b), used with Equation (5), will give a well-balanced design entirely safe with respect to failure by dishing. Equations (2), (3), and (4) should be used when close clearances or other special requirements make a balanced design impossible.

In connection with the application of this paper to the design of eye-bars it has been brought to the writer's attention that Section 411 of the A. R. E. A. Specification for Steel Railway Bridges has been changed to recommend a maximum net width-to-thickness ratio at the pin-hole of 8 instead of 12.

The writer wishes to express his appreciation to Mr. Hussey for his courtesy in discussing the paper.

NOTE.—The paper by Bruce G. Johnston, Assoc. M. Am. Soc. C. E., was published in March, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1938, by Harold D. Hussey, M. Am. Soc. C. E.

¹¹ Asst. Prof. of Civ. Eng. and Asst. Director in charge of Research, Fritz Eng. Lab., Lehigh Univ., Bethlehem, Pa.

^{11a} Received by the Secretary January 10, 1939.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

NATURAL PERIODS OF UNIFORM CANTILEVER BEAMS

Discussion

BY LYDIK S. JACOBSEN, ESQ.

LYDIK S. JACOBSEN,¹⁹ Esq. (by letter).^{19a}—As Mr. Hirashima states, tables with corrections for periods of buildings due to various types of foundations might be computed so as to make the designer's job of predetermining natural periods easier. However, for most of the practical designs, engineering judgment, fortified by qualitative knowledge of the relative importance of the different factors entering into the problem, will lead to results that are sufficiently accurate for design purposes. It must not be forgotten that the practical question relates to stresses in the building, and that it is only indirectly that natural periods of a structure merit study. Since the ground periods likely to be present in an earthquake are not definitely known, it is not necessary to predetermine the natural periods of the structure very accurately. The study presented in this paper attempts only to give quantitative backing for a sound evaluation of general tendencies; extensions from the specific examples given in the paper should rely chiefly on engineering judgment.

Mr. White's discussion of the paper introduces the question of the magnitude of error arising from considering continuous, rather than discontinuous, mass and rigidity distributions. He shows that for a highly simplified case satisfactory agreement is obtained if the structure consists of two or more stories. It should be pointed out, however, that the method used by Mr. White is definitely limited to a consideration of shear distortions and ground translations.

In the writer's opinion the effect of flexure of the building as a whole, as well as the effect due to ground tilting, may be estimated for most practical cases if one makes use of existing experimental data and is guided by a rational theory. In this case shear alone, or shear with estimated ground translation, becomes the only calculation that can reasonably be demanded by practice.

NOTE.—The paper by Lydik S. Jacobsen, Esq., was published in March, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1938, by K. Bert Hirashima, Esq.; and December, 1938, by Merit P. White, Jun. Am. Soc. C. E.

¹⁹ Prof. of Mech. and Civ. Eng., Stanford Univ., Stanford University, Calif.

^{19a} Received by the Secretary January 9, 1939.

For this purpose a schedule form of calculation involving successive approximations, and first given by H. Holzer, enables the designer to calculate the natural periods of non-uniform buildings with considerable accuracy and ease. As an example of Holzer's method a torsional vibration problem has been solved by Professor Timoshenko.²⁰ The writer has used a slight modification of this method for dealing with non-uniform buildings, and it can be said that the amount of work is very small, even for a building of many stories, when it is compared to the customary static calculations for the idealized framework of a building.

²⁰ "Vibration Problems in Engineering," by S. Timoshenko (Second Edition), p. 264.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

LABORATORY INVESTIGATION OF FLUME TRACTION AND TRANSPORTATION

Discussion

BY HUNTER ROUSE, ASSOC. M. AM. SOC. C. E.

HUNTER ROUSE,⁶⁶ ASSOC. M. AM. SOC. C. E. (by letter).^{66a}—A comprehensive treatment of sediment transportation is not an easy task, for at the present time the subject is in a pronounced state of flux. Moreover, for want of more conclusive experimental evidence, every man's opinion seems to differ from that of the next. The writer, therefore, confines his remarks to one portion of the author's mathematical derivation and to several phases of the general problem which warrant amplification in the light of research results not mentioned in the paper.

Perhaps the safest way to arrive at a general equation for tractive force is to follow the development of the energy equation for gradually varied flow. The basic expression,

$$E = \frac{V^2}{2g} + y + h_0 \dots \dots \dots (91)$$

is usually differentiated with respect to distance in the direction of flow:

$$\frac{dE}{dx} = \frac{d\left(\frac{V^2}{2g}\right)}{dx} + \frac{dy}{dx} + \frac{dh_0}{dx} \dots \dots \dots (92)$$

The term at the left then represents the local rate of change of total head. Designating a downward slope as positive, $-\frac{dE}{dx}$ will equal S_e , the slope of the energy line. For want of accurate knowledge as to the rate of energy loss in gradually varied flow, this slope is necessarily assumed to be the same as that

NOTE.—The paper by Y. L. Chang, Esq., was published in November, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1938, by Hans Kramer, M. Am. Soc. C. E.; June, 1938, by Messrs. E. W. Lane, and Joe W. Johnson; September, 1938, by Messrs. J. E. Christiansen, and W. H. Huang; and October, 1938, by Lorenz G. Straub, Assoc. M. Am. Soc. C. E.

⁶⁶ Associate Hydr. Engr., Cooperative Laboratory, Soil Conservation Service; Asst. Prof. of Fluid Mechanics, California Inst. of Technology; Pasadena, Calif.

^{66a} Received by the Secretary December 1, 1938.

for uniform flow at the same mean velocity and depth;⁶⁷ thus,

$$S_e = \frac{V^2}{C^2 y} \dots \dots \dots (93)$$

The error involved in this assumption is probably not serious, in particular if the flow is in the tranquil state. Such an assumption as to the rate of energy loss is tantamount to making the boundary shear equal to that in uniform flow for the same values of y and V , because, regardless of how the rate of energy dissipation may be distributed over the flow section, the resulting shearing stresses are transmitted undiminished to the fixed boundary. It then follows that,

$$\frac{dE}{dx} = -S_e = -\frac{V^2}{C^2 y} = -\frac{T}{wy} \dots \dots \dots (94)$$

The second term of Equation (92), showing the rate of change of velocity head, is written in terms of depth and mean velocity as follows:

$$\frac{d\left(\frac{V^2}{2g}\right)}{dx} = \frac{d\left(\frac{q^2}{2gy^2}\right)}{dx} = -\frac{q^2}{gy^3} \frac{dy}{dx} = -\frac{V^2}{gy} \frac{dy}{dx} \dots \dots \dots (95)$$

The third and fourth terms may be combined to yield the slope of the water surface,

$$\frac{d(y + h_0)}{dx} = -S_w \dots \dots \dots (96)$$

while the fourth term alone represents the slope of the channel bottom:

$$\frac{dh_0}{dx} = -S_0 \dots \dots \dots (97)$$

Upon introducing these several equivalent values, the differential equation may be rewritten with the intensity of boundary shear as the dependent variable:

$$T = wy \left(\frac{V^2}{gy} \frac{dy}{dx} - \frac{dy}{dx} + S_0 \right) \dots \dots \dots (98)$$

Equation (98) is the general expression for tractive force in gradually varied flow, and is recommended in place of the author's Equation (15).

In the case of non-uniform flow in a horizontal channel—under which condition the author's experiments were conducted, $-S_0$ is zero and $\frac{dy}{dx}$ becomes equal to $-S_w$; whereupon.

$$T = wS_w \left(y - \frac{V^2}{g} \right) \dots \dots \dots (99)$$

Equation (99) should replace the author's Equation (17).

Since the author so evidently sought to present an inclusive résumé of past research in this field, it is unfortunate that he failed to include a very valuable

⁶⁷ "Hydraulics of Open Channels," by B. A. Bakhmeteff, M. Am. Soc. C. E., Eng. Societies Monograph Series, McGraw-Hill Book Co., 1932, p. 25.

paper by A. Shields⁶⁸ which successfully answers more than one troublesome question touched upon in the paper. In this investigation it was shown that the relative magnitudes of tractive force and the resistance to motion of uniform sediment grains could be expressed dimensionlessly in the form,

$$\frac{T}{(\sigma - \rho) w D}. \quad (\text{The writer presumes, despite evidence to the contrary, that}$$

the author's symbols for specific gravity of sediment and fluid, σ and ρ , have their usual dimensionless significance, as distinguished from w , the specific weight of water.) This ratio, therefore, represents a fundamental parameter in bed-load movement, subject to the possible inclusion of additional factors for shape and grading of the material. Through comparison with Nikuradse's analysis of flow in artificially roughened pipes,⁶⁹ it was then proved that the beginning of movement from a leveled bed of uniform grains should depend

upon a single additional parameter, $\sqrt{\frac{g T D}{w \nu}}$. This parameter, furthermore, is proportional to the ratio of grain diameter to boundary-layer thickness, and as such has already been used to co-ordinate the measurements of pipe resistance for various conditions of relative roughness.⁷⁰ In other words,

$$\frac{T_0}{(\sigma - \rho) w D} = \varphi\left(\frac{D}{\delta}\right) \dots \dots \dots (100)$$

Shields made careful measurements of initial movement on beds of pulverized amber, lignite, granite, and barite, each of uniform size, and plotted his results dimensionlessly—together with those of other investigators using sand—as shown in Fig. 23. Despite considerable variation in specific gravity, diameter, and conditions of flow, the plotted values are seen to follow a single functional trend. At the left the laminar boundary layer still envelopes the particles on the bed, and the initial motion is not dependent upon turbulence; conditions of flow are, therefore, similar to those along a smooth boundary. When the particles begin to protrude through the laminar layer, initial effects of boundary roughness are noticeable, and the curve gradually approaches a minimum value. An upward trend begins when particles become sufficiently large to disrupt the laminar flow around them. An asymptotic limit is finally reached when the boundary layer no longer exists, owing to the turbulent wake produced by bed roughness of relatively great magnitude. The form of the curve closely approximates that resulting from Nikuradse's measurements on rough pipes.^{69, 70}

⁶⁸ "Anwendung der Aehnlichkeitsmechanik und der Turbulenzforschung auf die Geschiebebewegung," by A. Shields, *Mitteilungen der Preussischen Versuchsanstalt für Wasserbau und Schiffbau*, Heft 26, Berlin, 1936. Translations are on file at the Engineering Societies Library, 33 West 39th Street, New York, N. Y.; National Hydraulic Laboratory, Washington, D. C.; and the U. S. Waterways Experiment Station, Vicksburg, Miss.

⁶⁹ "Strömungsgesetze in rauen Röhren," by J. Nikuradse, *VDI Forschungsheft* 361, 1933. See "Modern Conceptions of the Mechanics of Fluid Turbulence," by Hunter Rouse, *Transactions, Am. Soc. C. E.*, Vol. 102 (1937), p. 485.

⁷⁰ "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, *Eng. Societies Monograph Series*, McGraw-Hill Book Co., 1938, p. 251.

Although the problem of initial movement has also been approached in terms of boundary-layer thickness by W. W. Rubey¹¹ and others, Shields' analysis appears to provide the solution sought by Professor Chang (compare with Figs. 6 and 7). Shields further noted that the relative dimensions of D and δ also govern the form of the initial bed irregularities. Particles moving within the laminar layer show a pronounced tendency to form ripples. Sediments slightly disrupting the boundary layer form "scales"—that is, longer, lower ripples. The ratio of length to height of the bed undulations increases steadily with the parameter, $\frac{D}{\delta}$, until finally the entire surface of the bed

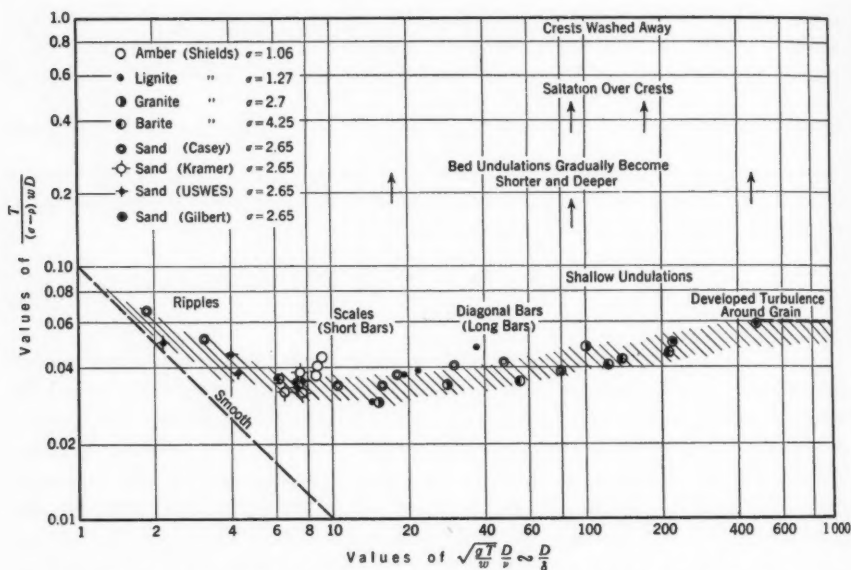


FIG. 23.—DIMENSIONLESS PLOT OF INITIAL BED-LOAD MOVEMENT FOR SEDIMENT OF UNIFORM SIZE, ACCORDING TO A

moves fairly uniformly as the curve of initial motion approaches its horizontal asymptote.

In Fig. 23 values of the bed-load parameter greater than the minimum evidently represent stages of movement beyond the initial phase. Shields, therefore, used this parameter in the form, $\frac{(T - T_0)}{(\sigma - \rho) w D}$, to develop an equation for rate of transport. The remaining dimensionless parameter is composed of the ratio of weight discharges of sediment and water, the specific gravities of sediment and water, and the slope. In order that the viscosity may have no appreciable influence upon the movement, the analysis is restricted to conditions in which the ratio of height to length of the bed undulations is low. Measured rates of movement for the five different types of material, plotted in Fig. 24, are

¹¹ "The Force Required to Move Particles on a Stream Bed," by William W. Rubey, U. S. Geological Survey, Professional Paper 189-E, 1938.

seen to follow the equation,

$$\frac{G}{Q S} \frac{\sigma - \rho}{\rho} = 10 \frac{T - T_0}{(\sigma - \rho) w D} \dots \dots \dots (101)$$

over nearly a thousand-fold range on either scale. The scattering of points, so characteristic of experimental results of this nature, is attributed to long-period cycles of bed movement; in addition, points for shooting flow lie somewhat

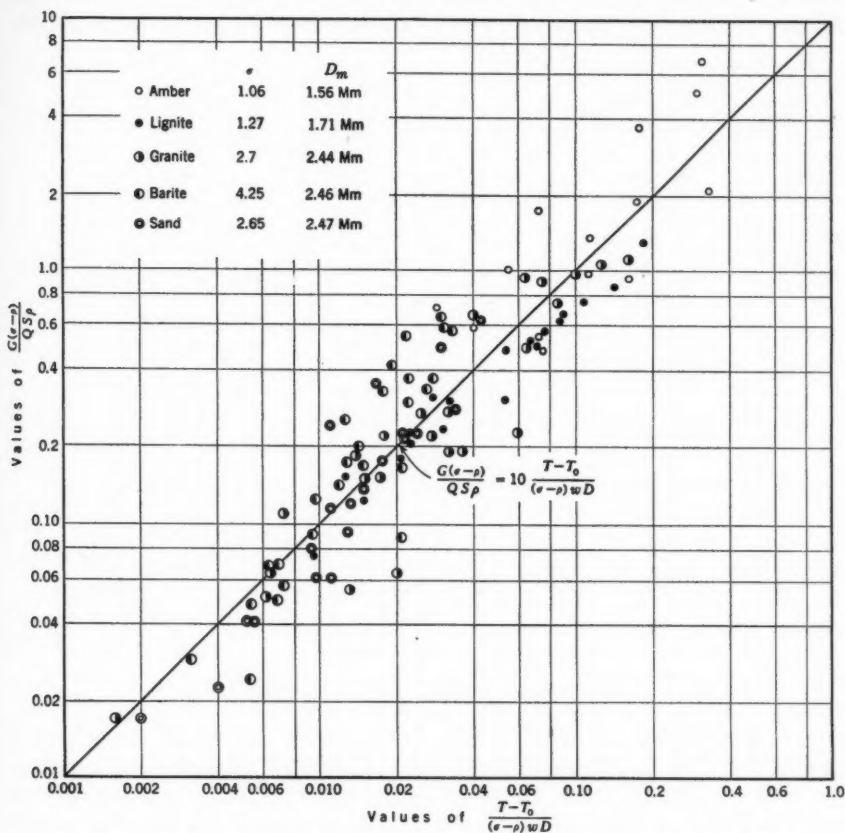


FIG. 24.—DIMENSIONLESS PLOT OF BED-LOAD TRANSPORTATION FOR SEDIMENT OF UNIFORM SIZE, ACCORDING TO A

higher than the others. This relationship is not to be considered an ultimate statement of bed-load transport, for the influence of shape and grading of the sediment grains still remains to be determined.

With regard to the author's treatment of suspended load, the writer would call attention to the fact that the validity of Equation (51) has been proved experimentally.⁷² On the other hand, it remains to be seen whether the magni-

⁷² "Experiments on the Mechanics of Sediment Suspension," by Hunter Rouse, *Proceedings, 5th International Congress for Applied Mechanics*, 1939.

tude of ϵ determined from the velocity gradient in turbulent flow will correspond numerically to that for the sediment concentration at the same elevation. Otherwise, abundant evidence of a logarithmic velocity distribution in wide channels would seem to justify the form of the suspended load function published by the writer.³²

The author attempts to integrate a simplified distribution function in order to express the total suspended load carried by a given flow—the total load, that is, of a given particle size. Obviously, in the case of a graded sediment, the process must be repeated for every different size. It is possible to simplify

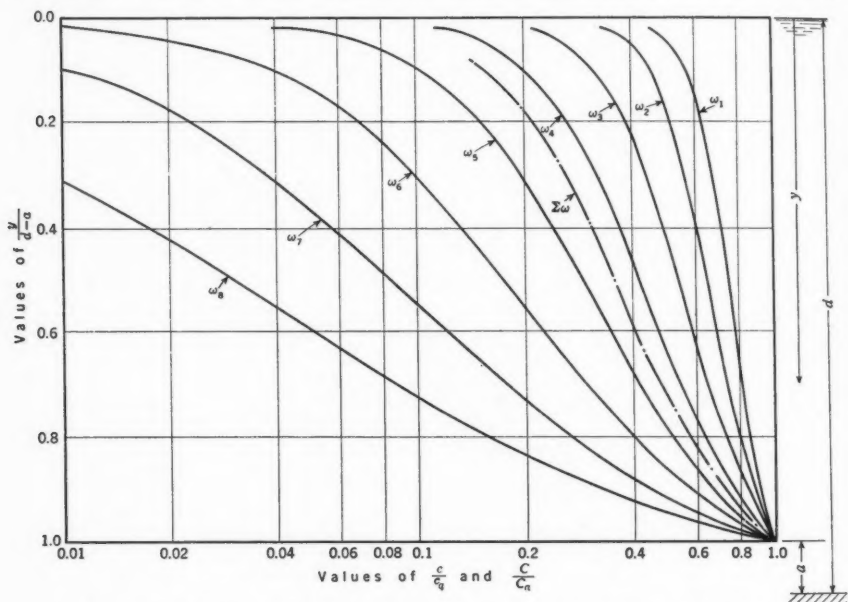


FIG. 25.—RELATIVE DISTRIBUTION OF TOTAL SUSPENDED LOAD IN THE CASE OF A NORMALLY GRADED SEDIMENT AT LEVEL a

this procedure for specific conditions in a manner which may also prove to have general application. If the fall velocities of the various sizes of sediment at the level, a , follow a normal distribution curve (that is, linear when plotted on logarithmic probability paper), the ratio of total concentration, C , at any higher level to that at the level, a , will follow closely the distribution curve for the particle size having the geometric mean fall velocity. This relationship is shown schematically in Fig. 25 for the distribution function of Equation (54), although it should hold equally well for any other analytic relationship between ϵ and y .

Evidently, the distribution of $\frac{C}{C_a}$ (shown as a broken line in Fig. 25) is practically the same as that of $\frac{c}{c_a}$ for the geometric mean value of ω . Graphical integration according to the assumed (or measured) velocity distribution curve would then be a relatively simple matter.

³² *Transactions, Am. Soc. C. E.*, Vol. 102 (1937), p. 463.

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DISCUSSIONS

DESIGN OF PILE FOUNDATIONS

Discussion

BY D. P. KRYNINE, M. AM. SOC. C. E.

D. P. KRYNINE,²⁹ M. AM. SOC. C. E. (by letter).^{29a}—The design of a pile foundation (as of any other foundation) should be accompanied by an estimation of its probable settlement. Hence the design should include two determinations: (1) Stresses (pressures) on each pile; and (2) the probable settlement of the structure from these pressures. Mr. Vetter gives a method of finding pile pressures, but does not solve the second part of the problem. Apparently, some experimental work is needed; for instance, driving and loading of experimental piles, or some other field testing.

Methods of Finding Pile Pressures.—An historical outline of different methods of finding pressures under a pile foundation has been prepared by Professor A. Agatz³⁰ which, in conjunction with this paper, represents a comprehensive treatment of how the methods of design have been developed to their present stage.

The method of Culman, presented before 1870, was to break the resultant of forces acting at a pier in three components using an "auxiliary" force direction. Each component was the sum of pressures in all the piles running in that particular direction; and the stress in an individual pile was found by dividing that component by the number of corresponding piles. The well-known office method of trapezoidal pressure distribution under an eccentrically loaded retaining wall followed. Another method is to subdivide the vertical load proportionally to the distances between the piles.

These approximate methods have been replaced by methods in which the mathematical theory of elasticity is used. In his "Acknowledgments" Mr. Vetter gives several references to these methods; and his list might be amplified

NOTE.—The paper by C. P. Vetter, M. Am. Soc. C. E., was published in February, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1938, by Messrs. Hibbert M. Hill, and Odd Albert; June, 1938, by Messrs. August E. Niederhoff, A. A. Eremin, and Jacob Feld; September, 1938, by John M. Coan, Jr., Jun. Am. Soc. C. E.; and January, 1939, by A. Agatz, Esq.

²⁹ Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

^{29a} Received by the Secretary January 6, 1939.

³⁰ Der Kampf des Ingenieurs gegen Erde und Wasser im Grundbau, 1936, p. 166 *et seq.*

by the addition of Krey,³¹ Jakobi,³² and Mügge.³³ Mr. Vetter's assumptions and his general approach to the solution of the problem are similar to procedures used by previous investigators, as Mr. Vetter himself states.

The Rôle of the Earth Mass Under-Estimated.—The writer is not prepared to offer a theory of his own concerning the design of pile foundations. Therefore, he limits himself to expressing his general opinion on the methods which use elastic theories. It is assumed that the tips of the piles are in a steady position. This is because the piles are restrained or because there is a hinge either at the tip of the pile or at a certain depth as, for instance, at two-thirds the length of the pile. The former case is represented when piles reach rock or similar solid soil and the latter corresponds to friction piles. The writer doubts that, in a general case, the tips of the piles do not move. Stresses in the earth mass are not taken care of, except "passive resistance," which is sometimes mentioned. Deformations in the earth mass are not considered. In other words, the fact that an earth mass with piles driven in it in groups and subjected to lateral motion acts as a unit is neglected.

Isolated Piles Subjected to Lateral Forces.—The behavior of a group of piles under a foundation has not yet been investigated experimentally from this point of view; and some opinion thereon may be formed only by analogy with experimental data concerning the behavior of isolated piles and small groups of them. If an isolated vertical pile is subjected to the action of a horizontal force, it is bent, the "zero-point" (Z , Fig. 21) being located approximately at the third point of the restrained part, h . The accurate position of Point Z depends on several variables, among them being the elastic constants of the material of which the pile is made, and the earth material. This action would be the same whether or not the tip of the pile reaches rock, but it is not known very exactly what may happen if a heavy vertical load, P , is applied to a friction pile. It may be guessed that the lateral deflection, δ , will be smaller than without Load P .

If the horizontal force, H' , as shown in Fig. 21, is removed, the pile will regain its original (straight) shape, but will not be vertical any more due to a non-reversible (plastic) deformation of the soil. An analogous phenomenon probably occurs in the case of a pile foundation, and it may be that these plastic deformations are much greater than the elastic deformations considered in present methods of design.

The anchor of a bulkhead is bent as shown in Fig. 22. This gives some indication on the possible behavior of battered piles. There still will be a hypothetical hinge higher than the tip of a battered pile. The writer welcomes the idea of using battered piles for resisting horizontal forces and emphasizes the necessity of making pulling tests with such piles.³⁴

Reinforced Earth.—The writer believes that when there are many piles under a structure subjected to the action of lateral forces, the stressed condition

³¹ Erdruck, Erdwiderstand und Tragfähigkeit des Baugrundes, Berlin, 1932, p. 162.

³² Ost. Wochenschr. öff. Baudienst, 1909, p. 340.

³³ Beiträge zur zeichnerischen Lösung technischer Rechnungsaufgaben. Dissert., Hannover, 1906.

³⁴ Transactions, Am. Soc. C. E., Vol. 102 (1937), p. 278.

becomes exceedingly complicated, and that a simplified, rather conventional method of design should be used. In a pile foundation the earth is "reinforced," something like reinforced concrete.³⁴

Piles Designed as Columns.—In the light of several assumptions by Mr. Vetter the writer wishes to discuss his opinion as to the effective length of hinged piles. Mr. Vetter states that if the pile is driven through loose material

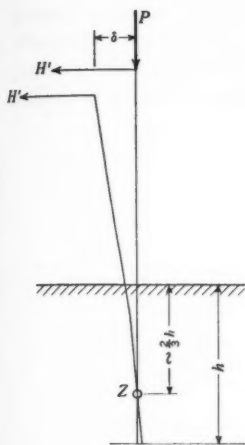


FIG. 21

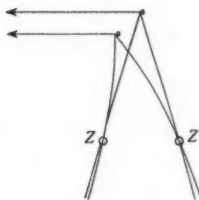


FIG. 22

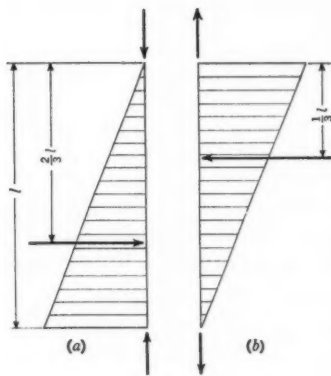


FIG. 23

to firm ground, the effective length of the pile will equal its length. This is not so, however, since the action of the "loose material" cannot be neglected and in reality is not neglected. To prove it, imagine piles 30 ft long standing vertically in the middle of the street, 3 ft apart, and carrying a substantial building. No engineer would dare to erect such a structure; however, in designing piles as columns exactly such a condition is considered. Cross-sections of the piles may be designed in this way. However, in computing the "effective length" some correction to the action of the adjacent soil must be introduced because in computing settlements the designer has no right to introduce arbitrary assumptions even if they are on the safe side. If he does so, his results will be in error.

Friction Piles.—Mr. Vetter and many other writers on this subject assume the triangular friction distribution along a friction pile (Fig. 23(a)). This rather contradicts some existing experimental data and field observations. From several "energy-penetration curves" taken during pile-driving, it may be concluded that the friction resistance during driving is rather uniform along the pile,³⁵ although this statement by no means can be generalized or extended, unconditionally, to the case of static loading. However, assume that the triangular friction distribution as shown in Fig. 23(a) is correct. Suppose, furthermore, that instead of compression, a friction pile is subjected to tension.

³⁵ *Proceedings, Soil. Mech. Conf., Vol. 3 (1936), paper H-16.*

For instance, in Fig. 9 this would be the case with Pile 2 if the wind should blow somewhat stronger. The friction would be reversed in this case, and, to be consistent, one should imagine also the reversed friction distribution (Fig. 23(b)). A question arises as to whether the effective length in this case is two-thirds the length of the pile, one third, or some other value.

Wind Action.—In Fig. 9 it is necessary to consider also the reversed wind action (smaller, perhaps). This would modify the computed stresses somewhat.

Conclusion.—Mr. Vetter's paper is a valuable step in the direction toward which the design of pile foundations is being developed. This is an appreciable contribution to the rather meager English literature in this field. The writer, however, hopes for a trend toward simplifying the existing methods of design.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

COST OF ENERGY GENERATION SECOND SYMPOSIUM ON POWER COSTS

Discussion

BY F. KNAPP, ESQ.

F. KNAPP,⁷¹ Esq. (by letter).^{71a}—Power is stated by Mr. Uhl to be one of the few commodities the cost of which has steadily decreased whereas the cost of other commodities has been doubled or trebled. This statement requires some explanation. Public utilities combine two characteristics. They serve to supply the population with essential needs, such as those provided for by power stations, gas-works and water-works, railways, tramways, mail, and similar undertakings. The other and essential characteristic is the impossibility, both of a technical and economic nature, of allowing free competition because of the tremendous importance of the necessary capital outlay and the requirements of the right of way. The monopolistic nature of the public utilities is practically unavoidable and explains the acrimonious discussions among those favoring either the private monopoly controlled by the Government, or the public monopoly. At any rate, any form of monopoly can be tolerated only as long as it serves the public interest. Experience in the United States, where the system of the privately owned utilities predominates, has shown the weak spots and the difficulties of an effective control, leading to rather unhealthy conditions. In the case of the public utilities owned by the Government, the management comes, daily and hourly, in close and sensitive contact with the public, thus submitting the undertaking to an extremely efficient control. The prestige and authority of this type of public utility often represent a political weapon of outstanding importance.

The danger of a monopoly arises from the removal of the natural law of supply and demand, thus offsetting completely the regulation of the cost of

NOTE.—This Symposium was presented at the meeting of the Power Division, New York, N. Y., January 20, 1938, and published in April, 1938, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: October, 1938, by Messrs. Joel D. Justin, Frank H. Mason, W. V. Burnell, Daniel W. Mead, A. G. Christie, William E. Rudolph, Louis Elliott, H. G. Gerdes, Leverett S. Lyon, John T. Madden, R. L. Thomas, Theodore B. Parker, Philip Sporn, L. N. McClellan, James C. Bonbright, R. Robinson Rowe, V. B. Libbey, and Edwin D. Dreyfus; and November, 1938, by R. H. Parsons, Esq.

⁷¹ With São Paulo Tramway, Light & Power Co., Ltd., São Paulo, Brazil.

^{71a} Received by the Secretary November 10, 1938.

the goods as established by competition. Making the assumption that the monopolist wishes to obtain a profit as great as possible (an assumption which is not far from the truth), the question arises as to just what level of cost will be necessary for him to reach his purpose. Choosing a high cost, the profit per unit sold rises, but the market probably becomes smaller. On the other hand, a large market permits him to reduce the cost per unit. Forced with this alternative, the monopolist certainly chooses the level that will make the product of cost per unit times the quantity sold, a maximum. This product being different for different cases, the monopolist will try to find this cost level by cut-and-try methods. Of deciding influence is the "elasticity" of the demand. It is now a well-established fact that the demand for power is extremely elastic; that is, a reduction of the cost to the ultimate consumer is accompanied by a considerable increase of the demand to such an extent as to make up for the loss caused by the original reduction. The significance of the elasticity of the demand for the fixing of the cost level makes it evident that the managements of every monopoly, and especially those of the electrical utilities, base their politics of cost upon a close (in most cases, statistical) knowledge of the coefficient of the elasticity of demand. Thus, in recent years, the cost of power is being reduced by economic reasoning, and has absolutely nothing to do with a philanthropic point of view.

As stated by Mr. Page, there is nothing bad in either public or private ownership of electrical utilities, provided the public interest is being served. There are too many examples to prove that this has not always been the case. Nothing is gained by arguing about a few mills per kilowatt-hour in the cost of the current as finally supplied to the customer. What is necessary is a broad consideration of the social and economic objectives of the electrical utilities and the need of placing the public interest above any other consideration.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

A THEORY OF SILT TRANSPORTATION

Discussion

BY MESSRS. GLENN W. HOLMES, AND HUNTER ROUSE

GLENN W. HOLMES,⁵⁶ JUN. AM. SOC. C. E. (by letter).^{56a}—Under the conditions established and on the assumptions made, the author's theory of silt transportation seems both logical and reasonable. In practice, however, the writer believes the conditions would rarely be met and the assumptions in general would not be found valid.

If the velocity in the center of a section exceeds that required for equilibrium as defined by the author, it will necessarily incline to scour. It does not follow, however, that the sides will shoal as the author claims. The scoured material is carried down stream and not directly to the sides, and the correspondingly reduced marginal velocity may not be slow enough to cause deposition of material coming from up stream.

In 1925, Fred C. Scobey and the late Samuel Fortier, Members, Am. Soc. C. E., wrote:⁵⁷

"* * *. It has been considered that for each depth of water there is a certain velocity below which silt will be deposited and above which silt will be eroded from the bed of the channel. In the opinion of the writers, two phenomena only slightly related are thus confused. The power of flowing water to maintain a movement of separate soil, sand, or gravel particles, has been confused with the power to break the bedding of a canal bottom and ravel off particles of that bed and transport them to places of lesser velocities where they may be deposited. In their [Messrs. Fortier and Scobey] opinion there is no sharp line of demarcation between the velocities that can no longer maintain silt in movement and those that will scour a canal bed. It is believed that there is a broad belt of velocities between these two 'critical' velocities, within which silt already loosened or brought in through a head-gate will remain in suspension while the bed nevertheless will remain undisturbed as

NOTE.—The paper by W. M. Griffith, Esq., was published in May, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1938, by Joe W. Johnson, Jun. Am. Soc. C. E.; October, 1938, by Messrs. George W. Howard, Harry F. Blaney, and E. W. Lane; and December, 1938, by Messrs. O. A. Faris, J. E. Christiansen, Samuel Shults, and Gerald Lacey.

⁵⁶ Associate Hydr. Engr., Soil Conservation Service, Washington, D. C.

^{56a} Received by the Secretary November 28, 1938.

⁵⁷ "Permissible Canal Velocities," by Samuel Fortier and Fred C. Scobey, *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 941.

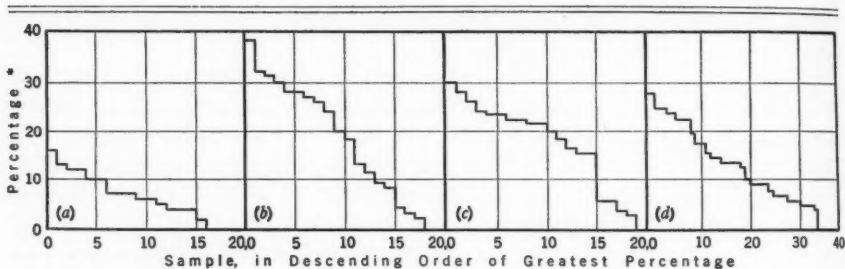
regards scour. It is easy to show the absurdity of accepting the laws of silting as giving the immediate answer to the laws of scour."

The inertia of particles in bed material, the cohesion, adhesion, mechanical interlocking, and other properties all contribute to a resisting force opposed to scouring—such resistance not having a counterpart in the forces opposed to transportation once the particles are in motion.

It would seem to the writer that clear streams and mud flows both deny the existence of definite critical velocities above which scouring occurs abruptly and below which sedimentation suddenly takes place.

Whether or not the author is correct in assuming such critical velocities or conditions of equilibrium, it is believed he attempts too great a step in applying his theory, based on a particular size range, to the measurement of the entire silt load of a stream. To be applicable to such a problem it is necessary to assume that there is a dependable relationship between total load and the part comprised of particles of sizes greater than 0.002 in. Table 9 shows the results

TABLE 9.—PROPORTION OF PARTICLES LARGER THAN 0.002 INCH IN SUSPENDED MATTER OF RANDOM SAMPLES OF THREE SMALL STREAMS



Curve	Description	Number of samples	PERCENTAGES OF TOTAL LOAD		
			Average	Maximum	Minimum
(a)	Coon Creek, at Coon Valley, Wis.	16	8	16	2
(b)	Coon Creek, at Stoddard, Wis.	18	19	38	1
(c)	Little La Crosse River, near Leon, Wis.	19	17	30	2
(d)	West Fork of Deep River, near High Point, N. C.	34	13	27	3

* Percentages of total suspended loads composed of particles larger than 0.002 in., arranged in order of magnitude.

of mechanical analyses made of silt samples collected by the U. S. Geological Survey for the Soil Conservation Service at four stream-gaging and silt-sampling stations. It is clearly shown that there was wide variation in the proportion of particles greater than 0.002 in., not only for different streams but also for different observations of the same stream. Although the samples were taken to represent the cross-sections at the time of observation, they were in other respects, without going into detail, essentially random samples. It should be

noted also that the larger silt particles made up only a small part of total loads.

The author states that the particles finer than 0.002 in. will not affect the bulk of the bed lining of channels in which loose granular material is being transported. This may or may not be true for channels or rivers in general; but if the theory is to be used to measure entire silt loads that may endanger the life of reservoirs or other slack-water sections, the particles larger than 0.002 in. are often found to play minor rôles in damages as may be seen in Table 10

TABLE 10.—SEDIMENT DISTRIBUTION IN LAKES AND RESERVOIRS

Lake, or reservoir	Location	Bulletin No. 524,* page:	SEDIMENTATION, IN ACRE-FEET		
			Delta deposits	Bottom-set beds	Total sediment
Lake Michie.....	Durham, N. C.	32	Not pronounced	395	395
Greensboro Municipal Reservoir..	Greensboro, N. C.	39	123	137	260
High Point Reservoir.....	High Point, N. C.	42	105	142	247
Concord Lake.....	Concord, N. C.	45	7.48	71.72	79.20
Spartanburg Reservoir.....	Spartanburg, S. C.	49	174	289	463
Lake Waco.....	Waco, Tex.	67	4 766	4 766
White Rock Lake.....	Dallas, Tex.	71	1 256	2 626	3 882
Guthrie Reservoir.....	Guthrie, Okla.	74	77	379	456
Boomer Lake.....	Stillwater, Okla.	77	0	170.93	170.93
San Carlos Reservoir.....	Coolidge Dam, Arizona	97	11 896	25 000	36 896

* "Siltling of Reservoirs," by Henry M. Eakin, U. S. Dept. of Agriculture, *Technical Bulletin No. 524*, July, 1936.

prepared from information presented by the late Henry M. Eakin⁵⁸ in 1936. Mr. Eakin wrote:

"It is of signal importance to systematic silt studies in that the volume of finer grained bottom-set beds, more frequently than otherwise, has been found to exceed the total volume of delta deposits and that, contrary to customary thought, the depletion of deeper reservoir storage space in the vicinity of the dam may not await the gradual approach of growing deltas but may, and in most cases does, begin at selective rates from the very beginning of storage."

It seems probable to the writer that no simple formula involving such factors as velocity and depth will be found to define silt loads of any particular size or range of sizes, because in addition to reasons already stated, those factors do not, and cannot, regulate or indicate the rate of debris supply, largely soil erosion, which is known to be a complicated process in itself.

HUNTER ROUSE,⁵⁹ Assoc. M. Am. Soc. C. E. (by letter).^{59a}—"Theory" is a word which has come to possess several distinct connotations. Among scientists, theories are generally based upon reasonably sound physical analysis; many such theories of past decades have become the natural laws of present-day science, after having stood the test of thorough experimental observation.

⁵⁸ "Siltling of Reservoirs," U. S. Dept. of Agriculture, *Technical Bulletin No. 524*, July, 1936, p. 7.

⁵⁹ Associate Hydr. Engr., Cooperative Laboratory, Soil Conservation Service; Asst. Prof. of Fluid Mechanics, California Inst. of Technology; Pasadena, Calif.

^{59a} Received by the Secretary January 10, 1939.

Although many engineers adhere to the scientific meaning of the word, in general practice numerous so-called theories vary from sheer hypotheses with neither analytical nor experimental foundation to concepts based upon either imperfect or totally erroneous methods of reasoning; as a result, more than one practical engineer has come to regard the word "theoretical" as the antonym of "actual." What is meant by "theory" in the present paper must be judged from the fact that it is based upon "two assumptions" and an "elementary law," the latter apparently having no more foundation than the two assumptions.

If Equation (1), the "elementary law," was developed on the basis of experimental data, it is a true empirical formula; if it was first formulated and subjected later to experimental check, it apparently began as a pure hypothesis. In neither case could it be considered to embody a scientific theory, and to regard it as a basic law is to presume that every possible experimental test had confirmed its validity. Since the author gives no reasonable physical justification for this initial statement, it would appear to be a purely arbitrary formula, which can be made to fit experimental data by adjustment of a variable coefficient and a variable exponent—as could other relationships of different mathematical form. The expression is really not as simple as it appears, for it embodies implicitly these two additional unknown functions.

Equation (2), again, may "follow from Equation (1)," but only if f bears the brunt of the change. This factor depends not only upon the shape of the cross-section, as the author states, but also upon the relative roughness of the bed and the mean velocity of the flow. For instance, f is surely not unity for a rectangular cross-section unless the width-depth ratio is large, as may be seen from any measured pattern of isovels. Perhaps f may be estimated empirically with sufficient accuracy for practical purposes; but if this factor can be "mathematically determined for any known type of cross-section," it is by a method as yet unknown to the writer. Equations (3) to (9) are subject to the same criticisms as Equations (1) and (2).

It would seem to the writer that the author's initial relationship could be classed with the dozen or more empirical formulas for bed-load transport, if used in the form

$$G = c_1 v = \frac{z v^2}{d^n} \dots \dots \dots (32)$$

to predict the amount of material carried per unit time and per unit width of flow section. It may be written in other forms, of course, by substituting from the equation for tractive force

$$T = w d S \dots \dots \dots (33)$$

and from the Chézy relationship

$$v = C \sqrt{d S} \dots \dots \dots (34)$$

This presumes, to be sure, that the flow is uniform and steady and that the channel is very wide, v and d then being mean values in accordance with the true significance of the Chézy equation. The author, however, recommends its

use for individual elements of a cross-section of irregular form, for cases of non-uniform and unsteady flow, and for the transportation of suspended load as well as bed load—indicating, moreover, that z and n will remain essentially constant under all these conditions. No bed-load investigator, to the writer's knowledge, has dared predict that his own formula would include even one of these departures from the normally assumed conditions of motion.

The writer does not wish to be interpreted as being against the empirical method of attack. The method is of extremely great importance to engineering practice, and in specific cases is even more likely to yield immediately useful results than methods which are purely rational. The problem is far more complex than the author's treatment would indicate, however, and the very complexity which prevents a rational solution also stands in the way of a general empirical treatment. With proper adjustment of coefficient and exponent, the equation in question will undoubtedly yield satisfactory results in many instances, and the author is only to be encouraged in studying the variation of these factors with boundary, flow, and sediment parameters. What the writer does protest is the acceptance of an apparently unfounded relationship as a cure-all for the thousand and one ills of the sediment problem, without thorough check. If the theory really has a sound physical basis, it is unfortunate that this was not demonstrated in the present paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

OBSERVED EFFECTS OF GEOMETRIC DISTORTION IN HYDRAULIC MODELS

Discussion

BY HERBERT D. VOGEL, M. AM. SOC. C. E.

HERBERT D. VOGEL,⁷ M. AM. SOC. C. E. (by letter).^{7a}—A rising vote of thanks is due Lieut. Nichols from the harassed brotherhood of hydraulic model testers, for he has laid low their ghost of despair and given them new courage with which to meet their difficulties. Fear engendered by ignorance is dispelled by the light of knowledge; hence, there should result from his paper a bolder and more intelligent approach to related problems. The analyses contained in the paper are worthy of serious study by all investigators of hydraulic phenomena. Although it is not desired to take material issue with Lieut. Nichols on any of the points raised by him, the following random thoughts, based on experience, appear to be in order.

Whether one likes it or not, it must be admitted that the ideal model is as much a figment of the imagination as the perfect fluid. Being forced at the start to use plain, every-day water in a model, and to keep it on this planet where the effects of gravity cannot be changed, one must relinquish any fond hope one may have harbored for the achievement of true dynamic similitude. It is not meant to imply by this statement that an investigator should don the eternally dark cloak of a defeatist; on the contrary, he must assume the attitude of an opportunist, ready to meet each newly developed situation squarely, anxious to try the untried, and determined to leave no stone unturned in his search for the ultimate truth. Above all, he must face facts as they are instead of as he would like them to be. There is no place in research for wishful thinking. The model is a handy, if relatively expensive, tool for determining, in advance, the probable effects of proposed changes in a prototype river or harbor. It should be expected to reveal such effects with accuracy and clarity. By a process of verification it can be tested as to trustworthiness, and, in this,

NOTE.—The paper by Kenneth D. Nichols, Jun. Am. Soc. C. E., was published in June, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1938, by Herbert W. Ehr Gott, Esq.; and November, 1938, by E. H. Taylor, Jun. Am. Soc. C. E.

⁷ Capt., Corps of Engrs., U. S. Army; Instr. The Army Engr. School, Fort Belvoir, Va.

^{7a} Received by the Secretary November 22, 1938.

all surety and hope repose. Although it is true that individual forces and accelerations of the prototype may fail of exact reproduction in the model, it is possible, nevertheless, to design and operate the latter so as to make it duplicate known effects. Thus, by "cutting and trying," by varying the variable conditions, by adjusting as the need directs, and by resorting to necessary distortions, the investigator can overcome dissimilarities engendered by the use of water and a garden-type of gravity. Perhaps he cannot determine the individual effects of each and every force, but—what is more important—he can come to an evaluation of resultant effects. If the model has been tested faithfully as to its capability to reproduce past occurrences, there is no reason to suppose that it will fail to predict the future with an equal degree of accuracy.

In general, allowing for differences in materials comprising beds and banks, every alluvial river may be considered as a rough model of every other. Traction in each must be sufficient to move the bed materials down stream; otherwise, there would be no river. If depths are less in the smaller rivers, slopes must be correspondingly greater; and, similarly, unless slopes are greatly exaggerated there must be definite increases in relative depths. Of course, there are definite limits to the slopes that may be obtained. Consider, briefly, the Elbe River, in Germany, as compared to the Lower Mississippi River, slopes of the former being greater on an average of from 25 to 70 per cent. If one may look upon the former as an approximate model of the latter, one should expect to find an exaggeration of the vertical scale. The horizontal scale, based on bankful measurements, is about 1 : 7; the vertical scale, similarly considered, is about 1 : 4. This indicates that a vertical exaggeration of about 1.75 has been applied by Nature herself.

Referring to Lieut. Nichols' statement that models of the Elbe River have been built to horizontal scales of 1 : 200 and 1 : 500, these values would correspond to scales of 1 : 1 400 and 1 : 3 500 for the Mississippi River. The vertical scale of the larger Elbe model (1 : 40) produced an exaggeration, in depths, of 5. Considering that Nature had already applied an exaggeration of 1.75, the total exaggeration was 5×1.75 , or 8.75. For a model of the Mississippi River, with a horizontal scale of 1 : 1 400, this would permit the selection of a vertical scale as large as 1 : 160, approximately. In the Elbe model, slopes were exaggerated by the factor, 8, in order to produce movement of the bed; but because of the warping that results therefrom in over-bank areas, this is no longer judged the best practice. Without supplementary slope to produce bed movement, it is probable that a vertical exaggeration of about 7 would have been necessary. This fact, translated to Mississippi River terms, would justify an exaggeration of more than 12, all of which tends to show that greater distortion may be tolerated in models of large, rather than in those of small, rivers. Extending the theory to small mountain streams, one can see that little or no distortion would be in order. It is fortunate, of course, that for such streams it is not needed, whereas the reverse is true for large rivers.

It is doubtful if any one should, or could, establish an ultimate permissible limit of distortion. Lieut. Nichols has wisely refrained from so doing, although he has indicated discrepancies that have been observed in models

greatly exaggerated as to depths and slopes. The entire problem resolves itself eventually into a question of money. If information is desired relative to the effects of proposed improvements in a certain reach of river and the facts are not obtainable by analytical methods, a model test is the obvious means available. The question of how much to spend is then pertinent. Certainly no test should be undertaken that will cost more than it can be hoped to save. Having determined the maximum expenditure permissible, it then remains to select suitable scale values. The horizontal scale is determined largely on a basis of costs; the vertical scale on a basis of depths that will be needed in the model. Distortion (or vertical exaggeration) is the result, and if an arbitrary limit is imposed, it may be that a model study will be precluded. If no worthy results are possible it would be better, of course, to abandon plans for a model then and there; but in almost every case some beneficial data may be expected. It is then necessary to weigh the estimated value of anticipated results against the cost thereof, in dollars and cents, and to arrive at a final decision on that basis.

A word of warning should be added in closing as an addition to the recommendations of Lieut. Nichols, all of which are concurred in. Models of spillways and similar structures should be made geometrically similar, in every respect, to prototypes. Distortion of any kind applied thereto results in a change of basic shape, and so alters frictional coefficients that direct comparisons are impossible. In an open-channel model, frictional differences are adjusted by the process of verification previously referred to, but in the case of spillway models it frequently happens that the prototype exists only in drawings or in imagination; hence, no check is possible. Since the heights of such structures are usually great with respect to length and widths, there is little temptation, if any, to exaggerate vertical dimensions. In case of an excessive length of crest, a sectional model will generally suffice.

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DISCUSSIONS

LATERAL EARTH AND CONCRETE PRESSURES

Discussion

BY D. P. KRYNINE, M. AM. SOC. C. E.

D. P. KRYNINE,³⁶ M. AM. SOC. C. E. (by letter).^{36a}—Much has been written in recent years concerning the determination of the vertical pressure in earth masses and very little about the horizontal pressure. This is the reason why the paper by Messrs. White and Paaswell is especially welcome. Their contribution offers an excellent opportunity to clarify the question of when the mathematical theory of elasticity may or may not be applied to the determination of the horizontal pressure in soils. Analytical procedures applicable to that science have now been developed to such an extent that average engineers feel themselves overwhelmed with complicated operations and page-long formulas; and some of the students in that field stop to ponder, and begin to ask themselves if they have gone too far with all this mathematics. After all, mathematics is nothing more than a simple engineering tool like a shovel or a pick and is to be used only to the extent necessary to serve its purpose. Perhaps, the time has come when investigators must simplify existing analytical procedures and search for some new and more direct methods of engineering analysis.

Continuity of Strains.—In order to apply the theory of elasticity to the determination of stresses within a body the strains must be continuous, and deformations must be extremely small. With these conditions in mind, the horizontal pressure against either a wall or a sheeting and bracing can be determined by using the theory of elasticity, but only if these particular structures or parts of structures do not yield under the action of that horizontal pressure. If they yield, the upper surface of the back-fill will settle, and there will be no continuity of strains.

Professor Terzaghi³⁷ has shown that when a retaining wall begins to move,

NOTE.—The paper by Lazarus White and George Paaswell, Members, Am. Soc. C. E., was published in September, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1938, by A. E. Cummings, M. Am. Soc. C. E.; December, 1938, by Messrs. Robert G. Hennes, Robert F. Legget, and Charles Terzaghi; and January, 1939, by Messrs. Jacob Feld, M. G. Spangler, and Raymond D. Mindlin.

³⁶ Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

^{36a} Received by the Secretary December 7, 1938.

³⁷ "A Fundamental Fallacy in Earth Pressure Computation," by Charles Terzaghi, M. Am. Soc. C. E., *Journal*, Boston Soc. C. E., Vol. 23, No. 2, April, 1936.

the horizontal pressure decreases and reaches a minimum (which is practically the value given by the Coulomb formula) at the moment of failure. Hence, the horizontal pressure determined by using elastic formulas must be greater than the pressure computed using Coulomb's formula. A rather rigid retaining wall, completely prevented from yielding, may fail, whereas a rather weak wall, but one that is permitted to move, may stand the strain quite satisfactorily. It may be concluded from this apparently paradoxical statement, that structures acted upon by the horizontal pressure should not be fixed against a slight lateral movement; and this restricts the possible field of application of the theory of elasticity to such structures.

The pressure distribution shown in Fig. 2 never occurs. This statement can also be proved by the principle of continuity of strains. In fact, strains in a sand mass can be continuous only if isolated particles are so pressed against each other that no mutual displacement of grains is possible, and if only elastic deformations occur. Mass acquires this condition when isolated particles are entirely restrained against movement. It is evident that particles at the surface of a sand mass are not so restrained. They can be moved freely, and, therefore, do not form a body in the sense used in mechanics. They are merely loose grains which can produce no horizontal pressure. Experiments by M. G. Spangler,³⁸ Assoc. M. Am. Soc. C. E., have revealed zero horizontal pressure at the top of a sand-gravel mass, which is quite consistent from the theoretical point of view.

When the back-fill is composed of clay (which may be considered as an elastically isotropic body) the situation is somewhat different. The horizontal pressure computed according to elastic theories is first negative (directed from the wall), the computations progressing from the surface of the ground, *A*, down to a certain point, *B*; then it is positive (toward the wall) down to a certain point, *C*; and negative again from Point *C* downward, as shown in Fig. 12 which is a simple free-hand sketch. Pressure measurements would show positive pressure along Distance *BC* only; and the appearance of a fissure at the top of the wall along Distance *AB* may be expected. In any event, Fig. 12 is not the same as Fig. 2, even in the case of a clay back-fill.

Poisson's Ratio.—Elastic formulas for determining stresses contain Poisson's ratio, μ , or its reciprocal, $m = \frac{1}{\mu}$. As used in elastic formulas, m is a constant.

The authors propose using two different values of Poisson's ratio—the smallest for the sheeting and the highest for the braces. In other words, it has been suggested to design the sheeting for one kind of soil, and the bracing for another, both kinds of soil being completely hypothetical. Such a suggestion would be entirely acceptable if it were certain that Poisson's ratio maintains a constant value during the process of loading; but there is no such certainty since experimental data to support such a statement are completely lacking.

Investigators in soil mechanics agree that the modulus of elasticity of a soil is not a constant value; it increases with the depth. It is only logical, then, to

³⁸ "The Distribution of Normal Pressure on a Retaining Wall Due to a Concentrated Surface Load," by M. G. Spangler, *Proceedings, International Conference on Soil Mechanics and Foundation Eng.*, Vol. 1 (1936), pp. 200-207; also, *Bulletin No. 140*, Iowa Engineering Experiment Station (1938), Iowa State Coll., Ames, Iowa.

think that Poisson's ratio also changes its value with the depth since, due to compression by the over-burden and to the formation of a "disturbed zone" under the loaded part of the earth surface, physical properties of an earth mass change with the depth. This is still another argument against the use of elastic formulas in the given case.

In general, the use of the term "Poisson's ratio" as applied to soils is questionable. The term may be applied to an elastically isotropic "continuum" in which there are no pores or voids; it represents the ratio of lateral strain to the axial strain in such a continuum. In a porous material the lateral deformation that appears as the result of an axial stress, changes the shape of individual grains and gradually fills the voids with matter, while the surface of the mass settles. In this case, the term "Poisson's ratio" loses its original meaning. German engineers prefer the use of a conventional term, *Querzahl*, or "transverse number," instead of the classical term, "Poisson's ratio."

Horizontal Pressure Due to the Weight of the Earth Mass.—The authors suggest that Equation (11) be used to compute the horizontal pressure due to the weight of the earth; and, that the coefficient, K , in this equation, "may receive a maximum value of 0.6 and a minimum value of 0.2. A rational method of designing sheeting and bracing would be to use the maximum value of K for the more rigid members of the supporting system and a much smaller one for the flexible sheeting." The writer under-

stands that the authors advance this rule as an empirical one based on practical experience. From a strictly theoretical viewpoint, however, Equation (1), to which the authors refer after introducing Equation (11), was developed for a semi-infinite, elastically isotropic, mass and cannot be applied when retaining walls, sheeting, and bracing are included in that semi-infinite mass.

Live Load.—The situation involving a live load acting at the boundary of the back-fill, at a certain distance from the wall, is different from that corresponding to earth pressure. The live load is applied when the wall has already moved and is in a rather steady position. Due to repeated applications of the live load the earth mass acquires elasticity. Furthermore, the weight of a live load is generally small in comparison with that of the back-fill. Thus it may be assumed that in the case of a live load elastic formulas hold. Obviously, this is true only so far as stresses at a certain depth are concerned since the horizontal pressure at the ground surface equals zero, as previously stated.

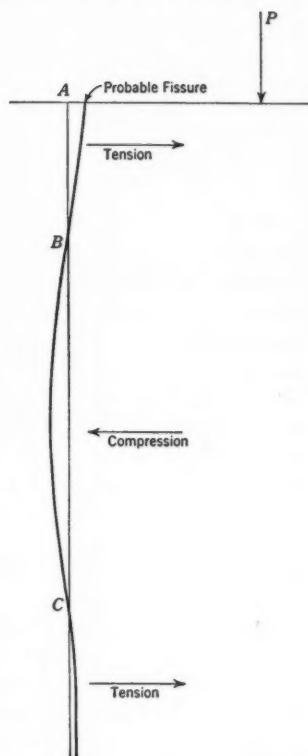


FIG. 12.—POSITIVE AND NEGATIVE HORIZONTAL PRESSURE

Spherical Co-ordinates.—Equation (12) representing the lateral pressure, F , is given in spherical co-ordinates, α , β and ρ . This is an excellent way to describe the stressed condition in the case of a single concentrated force, P , acting vertically at the boundary of the mass, since the entire problem is perfectly symmetrical about the vertical axis, OZ (Fig. 3(a)); but, in the case of

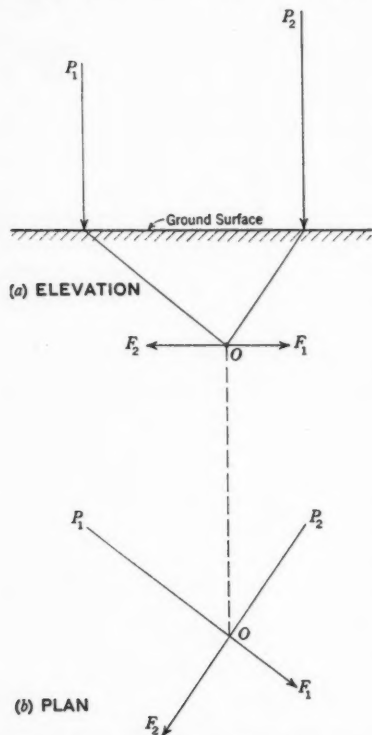


FIG. 13.—HORIZONTAL PRESSURE FROM TWO CONCENTRATED LOADS

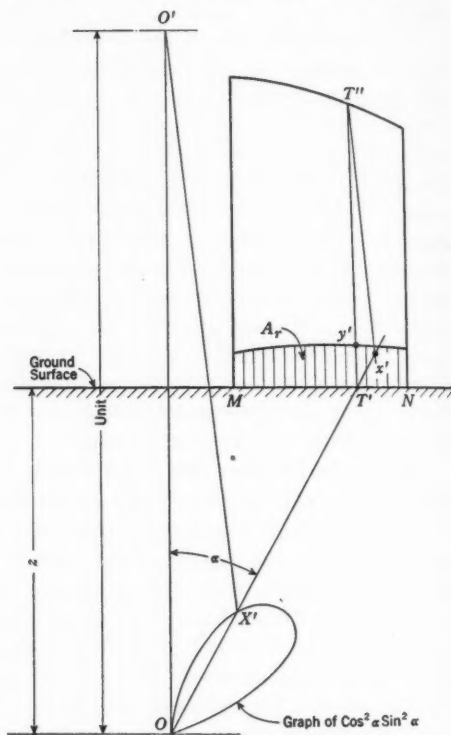


FIG. 14.—HORIZONTAL PRESSURE UNDER A STRIP LOAD

two or more loads acting at the boundary, there are difficulties in using spherical co-ordinates since the results given by Equation (12), for each of the given loads, cannot be added vectorially. To clarify this statement, Fig. 13 has been prepared. The maximum horizontal pressure caused by each of the two loads, P_1 and P_2 , acting at the ground surface, is confined in the vertical plane passing through the vertical line of action of the respective load and the given point, O . The two horizontal pressures intersect as shown in the plan (Fig. 13(a)) and are to be added geometrically.

Rectangular Loading.—It is to be borne in mind that Equation (14), and the text to Equation (22d), refer to the case of a rectangular loading, one of the sides of the latter being parallel to the retaining wall in question or, more accurately, to the plane normal to which the horizontal pressure in question acts (see drawing in Table 3). The authors rightly state that "the assumption of a single concentration is accurate" in the case of a rather small pier footing;

but, in the general case of a long rectangular loading, as shown in Fig. 3(b), this statement is no longer valid. It may be concluded from the paper that the authors themselves are of this opinion, but this fact should be emphasized. Mr. Spangler's experiments³⁸ destroyed the wrong assumption made by designers as to uniform distribution of pressure along the height of a wall if that pressure is caused by a surcharge. Messrs. White and Paaswell take a theoretical step in the same direction; and the writer appreciates greatly their plausible intention to contribute to the elimination from the engineering practice of an evidently erroneous assumption.

Strip Loading.—Instead of using analytical formulas as developed by the authors, graphical methods of solving this problem have been proposed.³⁹ In the general case of a non-uniform strip loading (Fig. 14) covering a section, MN , at the horizontal surface, of a semi-infinite elastically isotropic body, the horizontal pressure at Point O may be determined by tracing a vertical line, $OO' = 1$, and plotting along radial vectors, drawn from O , values of $\cos^2 \alpha \sin^2 \alpha = \frac{1}{4} \sin^2 2\alpha$. By joining the ends of the radii vectors, Locus $O'X'O$ is obtained. An ordinate, $T'T''$, of the area, $MT''N$, showing the non-uniformly distributed load, is "reduced" by drawing a straight line through points O and T'' , and joining the point, X' , of its intersection with the locus referred to, with Point O' . Drawing $T''x'$ (Fig. 14) parallel to $O'X'$, the value of the ordinate, $T'y' = T'x'$, of the "reduced" area, A_r , is obtained. The reduced area is then measured, by planimeter, or in some other way, and the value of the horizontal pressure is:

$$F = \frac{2}{\pi} \frac{A_r}{z} \dots \dots \dots (47)$$

in which z is the depth of Point O , under the ground surface. If the scale of the reduced area, A_r , in Fig. 14, is such that 1 in. represents a tons per sq ft vertically, and b ft horizontally, each square inch of that area would represent ab tons per ft. Then the stress intensity, F , would be measured in tons per sq ft. A somewhat different procedure proposed by the writer may be also used.⁴⁰

In the interest of a more accurate terminology, the "point load" in Fig. 4(a) should be termed "line load," since this is an infinitely long straight line at the ground surface loaded with \bar{p} units of weight per unit of length.

Horizontal Pressure and Pressure on a Retaining Wall.—Attention should be called to the fact that Pressures F , computed according to elastic theories, are stresses within the mass itself. Using the method of images, R. D. Mindlin,⁴¹ Jun. Am. Soc. C. E., has shown that the pressure on a retaining wall is not F , but $2F$.

Conclusions.—The main principle of this paper, in that it defends the application of the theory of elasticity to the determination of the horizontal pressure against sheeting and bracing, is questionable. The paper itself, however, is to be welcomed as a contribution to the field in which practically no mathematical study has been done in recent years.

³⁸ "Pressures Beneath a Spread Foundation," by D. P. Krynine, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), pp. 834-835.

⁴⁰ "The Stress Function and Photo-Elasticity Applied to Dams," by John H. A. Brahtz, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), pp. 1288-1295.

⁴¹ *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Vol. 3 (1936), p. 71.

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DISCUSSIONS

TRANSPORTATION OF SAND AND GRAVEL IN A FOUR-INCH PIPE

Discussion

BY MESSRS. R. L. VAUGHN, M. P. DUREPAIRE,
AND PIERRE F. DANIEL

R. L. VAUGHN,²⁷ M. Am. Soc. C. E. (by letter).^{27a}—It is encouraging to note that an important agency has begun to devote some of its attention and facilities to the sadly neglected subject of hydraulic dredging. In view of the magnitude of the sums expended annually for dredging, the nearly complete lack of scientific knowledge concerning the operation is astonishing but quite understandable by any one who has attempted to conduct investigations covering any phase of it.

The author states that the experiments were a part of an investigation into means whereby the capacity of pipe lines might be increased. A legend persists to the effect that, in the early days of hydraulic dredging, efforts were made to accomplish this end by inserting spiral baffles in occasional sections of pipe. These baffles were reported to be angle-irons of comparatively small size, but of considerable length (probably about 16 ft). Also, at one time, it was contended that spiral riveted pipe had an advantage in this respect. The idea, of course, was to impart a whirling motion to the mixture and thus prevent undesirable concentrations in the bottom of the pipe. Spiral pipe failed to substantiate its claims to superiority and baffles did not survive the initial trials.

The "apparent percentage of solids" is a convenient factor in dredging work since it furnishes a ready means by which the output of a dredger in operation may be calculated. It is a very treacherous factor for use when estimating the performance of a dredger in advance and is likely to give a misleading idea of the true properties of any mixture to which it is applied. Two banks of sand, of identical mechanical grading, may have substantially different percentages

NOTE.—The paper by George W. Howard, Jun. Am. Soc. C. E., was published in September, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1938, by Messrs. Fred. R. Brown, Joseph E. Montgomery, Elliott J. Dent, and David L. Neuman; and January, 1939, by Morrough P. O'Brien, M. Am. Soc. C. E., and R. G. Folsom, Esq.

²⁷ Cons. Engr., San Francisco, Calif.

^{27a} Received by the Secretary November 26, 1938.

of voids, depending upon the circumstances under which they were deposited. Obviously, if 1 cu yd of sand from each bank, in turn, is mixed with 9 cu yd of water, the result in both cases will be a mixture containing "10% of solids"; and yet the two mixtures may be substantially different. Similarly, the entire series of clays, from fluid muds to hard clays, may be regarded, for dredging purposes, as simply fine-grained matter mixed with more or less water. Any one of them, by the admixture of the requisite quantity of water, and with sufficient agitation, can be reduced to a fluid mud. If a soft mud bank is excavated by a dredger and pumped at the rate of 4 cu yd of water to each yard of mud, the only trouble with the mixture containing "20% solids" will be to prevent its escape from the fill into some navigable waterway where it is certain to irritate the United States District Engineer. On the other hand, if a stiff clay bank is excavated and pumped at the rate of 4 yd of water to one of clay, the resulting mixture of "20% solids" will irritate a number of people. Use of this and other redundant or misleading phrases, such as "bulk density" and "bulk specific gravity," should be avoided in engineering work. In the present case it is not clear whether the percentage was computed on the basis of a volume of sand with the voids filled with water, or on the basis of an equal volume with the voids filled by air.

Efforts to correlate observed phenomena on a false basis account for at least some of the troubles encountered by investigators attempting research on a subject which is at best most difficult and confusing. Two properties of any pumpable mixture are of primary importance; one is the weight, and the other is the grading of the particles. Another important factor, not solely a property of the mixture, is the ratio between the diameter of the larger particles and the diameter of the pipe through which the mixture is forced.

It has generally been assumed that a mixture of water and solid particles will act exactly as a liquid of equal weight, and that all the fundamental laws of hydraulics may be applied to it. This can be true only as long as the mixture is a uniform one; when material begins to settle in a pipe there is no longer one mixture but an infinite series of them grading from top to bottom; and when material begins to roll or drag along the pipe the area of the flowing stream is reduced, and most of the hydraulic laws no longer are valid. There is thought to be a critical velocity which will keep the mixture uniform, but no one has discovered any formula for it. Both the weight of the mixture and the grading of the particles seem to be involved. Where the larger particles are balls of clay or porous objects having a unit weight materially different from that of the solid matter in the mixture, the density of the particles will also be an important factor.

The effect of the grading is well known. It is notoriously easier to pump a mud than a sand, and there is a marked difference in pumping sands depending upon whether they are fine or coarse. On the other hand, any dredging man will comment feelingly upon the difficulty of pumping clay balls. However, no quantitative data on this subject have been presented to the profession.

The importance of the size of the pipe has received more recognition in recent years. Formerly, it was thought that so long as a velocity above the

critical one was maintained the output would be directly proportional to the weight of mixture, velocity, and area of pipe. With the advent of high-powered dredgers of large size it has developed that, all things considered, clay can be pumped through large pipe lines much more effectively than through small ones. The fact is known but not the fundamental laws governing it. Something other than mere friction loss of head seems to be involved.

Eventually, probably, some property of mixtures analogous to the viscosity of liquids will be evolved; and, probably also, something resembling the "effective size" and "uniformity coefficient" which have been evolved in connection with studies of the flow of water through soil. Whoever discovers these properties will have the privilege of naming them. The soil mechanics investigators have invented an imposing glossary to describe newly discovered and hitherto unsuspected properties of earths. Dredging specialists certainly have the right also to a jargon of their own whenever they can discover properties to which to apply the words. It is suggested that appropriate names for some of the new units would be "Gilmores," "von Schmidts," and "McMullins," in honor of some of the early pioneers. Surviving old timers would probably be unanimously in favor of naming a particularly objectionable characteristic after the late Alonzo P. Bowers.

The author mentions difficulties arising from the variable nature of the mixture in a dredger pipe line. In part, these difficulties are due to practical considerations involved in the operation of the dredger. Clear water must be pumped while "fleeting." When digging, the material will cave down on top of the suction at an irregular rate so that it is impossible even to approximate a constant feed. Some factor other than those due to operating conditions also seems to be involved. Occasions have arisen in which it was possible to introduce reasonably uniform material into a pipe line by controlled means at a constant rate. In some instances efforts were made to determine the loss of head. It was found impossible to maintain a constant mixture and, at the same time, attain any value approaching the maximum capacity of the set-up. Results of observations were about as satisfactory as have been all other observations concerning the hydraulic performance of an hydraulic dredger.²⁸

It is unfortunate that a motor was not used in the present experiments rather than a gas engine, since with an electric drive it would have been possible to determine the power used at various times. The author states that the velocity was first set at about 5 ft per sec and a solid concentration introduced. What happened to the velocity, and what steps if any were taken to bring it back to 5 ft per sec? How was the velocity determined in the first place?

After a state of equilibrium had been attained and some observations made, the velocity was increased but the rate of discharge of solids was kept constant. Again, how was the velocity measured, and how was the mixture controlled? From an inspection of Fig. 1 it is not clear how the procedure described could have been followed readily. On the other hand, with this arrangement it would have been easy to make a mixture of any predetermined consistency, and

²⁸ *Engineering News-Record*, June 16, 1921, p. 1035.

keep it circulating at any desired speed. After observations had been taken on one constant mixture at all velocities desired another mixture could have been made and observations again taken at predetermined velocities.

It was only necessary to make tests on gravel at velocities ranging from 5.5 to 8.5 ft per sec, which emphasizes the importance of the relation between size of particles and size of pipe. On a dredging scale it would be impossible to obtain satisfactory results when pumping this gravel at a velocity as low as 8.5 ft per sec.

It is not clear where the data pertaining to solid matter discharged, percentage of solids, and velocity of flow in pipes, came from (see heading, "Method of Transporting the Material: Sand"). The information is supplied in connection with Fig. 4 which shows a maximum velocity of only slightly more than 6 ft per sec, while the tabulated velocity is 8.88 ft per sec.

Mention is made of difficulties in determining the velocity distribution shown in Fig. 4. Evidently, the sampling device was used as a Pitot tube. Were the several pressure readings in each case reduced to feet of head of the particular mixture which existed at each of the several points where observations were taken? A small reaction plate could be calibrated and substituted for the Pitot tube, thus avoiding clogging troubles. Such a disk must have a considerable size relative to the size of the particles in the mixture, and in this case could not have been used for gravel as it would necessarily have been about as big as the pipe.

The author used feet of mixture rather than feet of water in the preparation of Tables 1 and 2. It is to be regretted that he did not follow this same correct procedure when drawing Fig. 6. He states that where material settled to the bottom of the pipe there was a greater loss of head than for slightly higher velocities because of increased pressures caused by the constrictions. The explanation is incorrect. Where constrictions occurred the local velocity, perforce, rose because the same volume was passing all points, and it was this increase in velocity which caused the increased loss of head. The pressure had nothing to do with it; and, in fact, there must have been a local drop in pressure at the constricted points, or the case for Bernoulli's theorem and all Venturi meters is weak indeed.

Changes observed through the windows are advanced as explaining the variations in values of f in Table 1. Aside from the rolling and jerking, what changes, if any, were observed? The rolling and jerking were reported to occur at rather definite velocities, whereas the variations in Table 1 appear to be quite regular throughout the entire range of velocities.

The opinion is expressed in the paper that it is impossible to obtain a formula for loss of head that is applicable to any but a specific gradation of material. In other words a separate formula is needed for each batch of dirt. The writer does not agree. He believes that when research has disclosed the inter-relationship between the essential variables, weight of mixture, gradation, specific gravity of particles, diameter of pipe, and velocity, a satisfactory formula can be written containing factors that reflect the influence of each variable.

A general criticism of the paper and of nearly all other investigations of the

kind is that the true nature of the problem is not recognized. The essential study is that of the flow of a mixture. When solid particles settle to the bottom of the pipe and begin to jump, slide, and roll, they no longer follow the law of flow of mixtures; nor does the water entrapped within them follow that law. The problem with respect to such accumulations is: What can a certain flowing mixture do in the matter of shoving material along the bottom of the pipe? The mixture itself is whatever remains actually flowing. The essential factors are the weight, gradation, velocity, etc., of the flowing stream. It is no more correct to include the rolling and dragging material than it would be to conduct dynamometer tests on a ship's model in a tank while the model was dragging an anchor behind it, and the results are about equally valuable.

From the accounts given, it is suspected that in these tests, at least at the lower velocities, the velocity was considered to be the result of dividing the total discharge by the area of the pipe, and that the percentage of solids was determined from the volume of sand and water in the sampling tank. Reflection will show that whatever sand was scraped into the tank had nothing to do with the quality of the mixture, and that the velocity should be that of whatever was flowing, not some quantity derived by including materials that did not flow and areas through which flow did not occur. It cannot be expected that the fundamental laws governing flow will be disclosed when these non-relevant items are reflected in the observations.

Figs. 7 and 8 appear much more concordant and orderly than most plottings of data of the kind, and it is possible that the attempt to evolve a formula was abandoned without due consideration.

Recently tests of transportation of sands in 2-in. and 3-in. pipes were conducted by Morrough P. O'Brien, M. Am. Soc. C. E., and Mr. R. G. Folsom, at the University of California.²⁹ Those tests form a valuable supplement to those described by the author.

The California experimenters point out that Stoke's law does not apply to particles of sizes frequently encountered in dredging practice, but, nevertheless, they find that, for sands, so long as the critical velocity is exceeded, the flow is a turbulent one, and therefore, as was to be expected from the laws of turbulent flow, the viscosity or density of the mixture is not a factor in the problem. They also found that, above the critical velocity, the loss of head expressed in feet of mixture was the same as it would be for clear water. All the available evidence, including that contained in the paper, tends to sustain these findings and the conclusion is important even if it is at variance with ideas previously held in dredging circles.

Messrs. O'Brien and Folsom also observed the performance of the pump and found that the pressure developed was not directly proportional to the weight of the mixture pumped. This is entirely in accord with the fact that Stoke's law does not apply to such mixtures. The general theory of centrifugal pumps indicates that the head developed with any liquid is constant, that is, that the pressure for any given discharge will be directly proportional to the weight of the liquid. When dealing with dredger pumps it has generally been

²⁹ Univ. of California Publications in Engineering, Vol. 3, No. 7, pp. 343-384.

assumed that the mixture would behave as a true liquid. In a previous discussion the writer questioned this assumption.³⁰

Under actual operating conditions on a dredger it is difficult if not impossible to maintain a constant mixture for a time sufficient to permit a determination of the weight of the mixture, the head developed, the power input, and the discharge. The observations at the University of California are the only ones known by the writer to bear directly on this problem. If the author should make any further tests it is to be hoped that he may be able to include the performance of the pump in his observations.

Messrs. O'Brien and Folsom concluded that the most important single property of a mixture was the settling velocity of the particles composing it. A turbulent flow is one consisting of a combination of linear flow and cross-flow. It is suggested that further investigation might reveal a relation between average velocity, cross-velocity, and diameter of pipe line, and further that it might be found that whenever the transverse velocity exceeded the settling velocity the flow became fully turbulent. In this manner it might become possible to predict, for any given pipe, the minimum velocity which would maintain turbulent flow for a mixture containing clay balls, cobbles, shells, and similar particles, the settling velocity of which could be determined.

As far as the writer is aware no observations have been taken of settling velocities of particles in any liquid but water. It might be found that the velocity of a clay ball, for instance, falling through a liquid mud was much lower than that falling through water, and, consequently, that the critical velocity for a mixture of mud and clay was less than that for clay alone.

Possibly the problem of transporting material in pipe lines is not one involving turbulent flow only. Experience suggests that the greatest rate of output may be attained when some of the material is carried in suspension, while, at the same time, some is also moved by being dragged or rolled along the bottom. This would involve two different types of flow in the same pipe at the same time and obstacles in the way of arriving at any laws governing the situation are obvious. If a mixed flow is the one which will transport the most material, reported troubles in determining the maximum or economic weight of the mixture would be explained since flow under this condition must be somewhat unstable.

All the author's formal conclusions and his remarks under the headings, "The Economical Velocity for Sand Transportation" and "Transferability of Experimental Results," are exceptionally sound and pertinent. They are endorsed unreservedly as far as they go.

The economical velocity for a dredger involves factors not mentioned by the author. Fig. 13 shows tests conducted with Columbia River sand on the dredgers, *Wahkiakum* and *Multnomah*. These are sister machines, with both suction pipe and discharge pipe 24 in. in diameter. It will be noted that as the pipe-line velocity increased, not only did the percentage of solids fall, but so did the output of the dredger, until the highest attainable velocity was approached. The answer to this seeming paradox lies in the dredger—not in

³⁰ Transactions, Am. Soc. C. E., Vol. 96 (1932), p. 651.

the pipe line nor in the flowing stream. By increasing the power used, the pressure and velocity can be raised to almost any reasonable extent; velocities of 35 ft per sec and working pressures of 125 lb are not unknown. On the suction side of the pump, however, the pressure is absolutely limited to 1 atmosphere. Nothing can be done to increase it, for which reason, in theory, the proper place for the pump is out on the ladder immediately behind the cutter.

This pressure of 1 atmosphere must do several things. It must overcome entrance losses, and friction losses, and must create velocity head. If the center of the pump is above the water-line it must lift all water pumped this additional distance. Most important, the excavated material must be lifted and fed into the pump. If a disproportionate fraction of the total available pressure is used to accomplish the former objects, little or nothing remains available to bring in pay dirt.

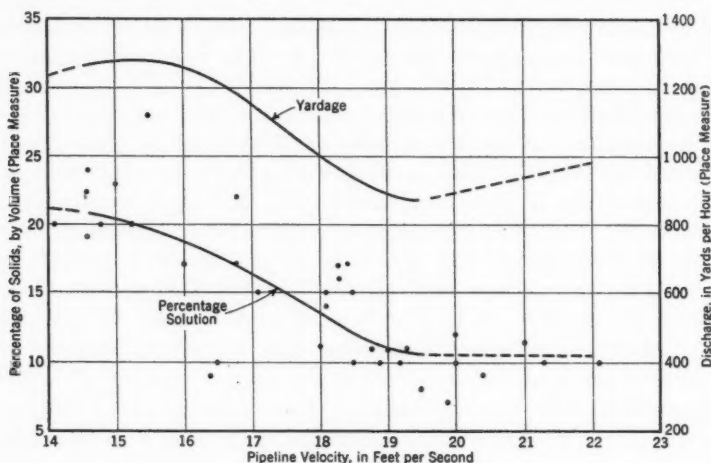


FIG. 13

The illustration is no isolated instance. The fact has been confirmed many times. On some electric dredgers the speed of the pump, and consequently the pipe-line velocity, may be varied at will, within rather wide limits. Power consumption is immediately visual through indicating watt-meters. By trial it has been found frequently that lowering the velocity would actually increase the output materially, not only when pumping sand but when pumping clay.

As far as the dredge operator is concerned, the economic velocity is the one that will get out the most material. The unit cost for "wear and tear" is about constant; unit costs for labor, over-head, interest, depreciation, etc., are inversely proportional to the output; and, although the unit cost for power may be slightly higher for the higher outputs the increase, if any, is seldom enough to offset the other gains. Of course, this is not the true problem of the economical velocity in the pipe line and the dredge operator spends his spare time wondering what would happen if the diameter of the suction pipe or the diameter of the discharge pipe were increased. Eventually he builds a new

machine in which the size of both is increased. After a comparatively brief operating interval it is decided that more power is needed, which is duly installed, and the operator is right back where he started except that he has a larger dredge.

Finally, the announced object of the experiments was to devise means whereby the flow of mixtures in pipes might be improved. Results of early-day experiments indicated that any device inserted in the pipe would constitute an obstruction causing more harm than good. Experience indicates that boosters are more effective than dredge pumps. For illustration, a combination of a 1 500-hp dredger pump and a 500-hp booster out in the line is likely to do better work than a 2 000-hp pump on the dredger without a booster. The reason the booster is effective is thought to be that it stirs the mixture.

Boosters are heavy machines, expensive to install; they require considerable labor for attendance, and are anything but mobile pieces of equipment. Their use is avoided wherever possible. In special instances, especially where electric power is readily available, it might prove feasible to fit a few sections of shore pipe with small motor-driven centrifugal pumps arranged to draw practically clear water from the top of the pipe line and inject it into the bottom again under considerable velocity. Devices of this kind, located about a half mile apart along a long pipe line, might be found to be surprisingly effective and to require much less power than one would be inclined to suppose. The writer has not heard of anything of the kind, but it would not cost much to try the idea.

M. P. DUREPAIRE,³¹ Esq. (by letter).^{31a}—In certain respects, sand transportation in dredge pipes is still in a crude, empirical state. A better understanding of the basic laws governing this technique is needed because great sums of money are spent each year in pumping sand mixtures through pipes in conditions which are often far from the best possible.

The paper by Mr. Howard is a welcome contribution to current knowledge on the subject in the wealth of experimental data and engineering remarks contained in it. Experiments with the same problem have been in process for several years in France, and a brief summary of the unpublished results thus far obtained should constitute a constructive discussion of Mr. Howard's paper. Tests on the flow of sand mixtures through water pipes were started in 1935, under the direction of engineers in charge of Nantes harbor. These tests were conducted in a steel pipe of 52 mm, inside diameter (2.05 in.). Loire River sand was used, screened so that the maximum grain size was 0.3 mm (0.12 in.). The volume of the voids was 43% and the specific gravity was 1.945 kg per cu decimeter (121 lb per cu ft).

The test layout was a closed circulating system operated with a centrifugal pump of adequate design. Two glass sections were provided for observing the conditions prevailing during each run, and the following conclusions were reached:

(1) When all the load is in suspension and within the range of concentration encountered in the experiments (to 40%), the loss of head (measured in feet

³¹ Ingénieur des Ponts et Chaussées, Nantes, France.

^{31a} Received by the Secretary December 14, 1938.

of the mixture under test) was the same as with clean water, regardless of the sand concentration, except at the stage when deposition is impending.

(2) For each concentration, a critical velocity was found corresponding essentially to that in which deposition occurs in a stage of decreasing velocities, this critical velocity being very approximately that at which the minimum loss of head occurs. With the grain size used in the Nantes experiments, the loss of head was never found to be less than that with clear water.

(3) It was concluded that, theoretically, jerking motion such as that described by Mr. Howard, and the well-known phenomenon of sudden blocking of the entire section, was a problem involving stability of flow conditions encountered with mixtures. This stability condition depends on the characteristics of the kinetic head. These phenomena can be avoided entirely by using a specially designed centrifugal pump. No jerking motion was observed in the glass sections of the Nantes experiments, whereas it seems impossible to avoid that type of motion using a gravity head such as that in the Howard experiments.

(4) The loss of head in the partial deposition phase appears to be a very complex phenomenon. It was found in the partial deposition phase that, for a constant concentration of sand, the loss of head is greater when the total discharge decreases. At the same time the height of deposited material increases. The deposit in a given run was of fairly uniform depth because no jerking motion was taking place. The same depth could be obtained in a fairly wide range of discharge with different concentrations of the flowing mixture. It was found that, for a given constant depth of deposit, the loss of head varied as the square of the discharge, whatever the concentration.

(5) Within the duration of each run, the deposition phenomenon did not seem to be a reversible phenomenon; that is, it requires higher velocity to pick up material than the velocities which prevailed when the material was deposited.

This means that in the partial deposition phase, the loss of head for a given discharge and a given concentration may have a very different value compared with the condition prevailing in accelerating or decelerating the flow. In the phase under discussion, the phenomenon cannot then be represented by a single curve, but covers a zone. In undertaking to study this feature more closely, it is essential to use a well designed pump.

The foregoing conclusions were deduced from the laboratory experiments. As far as actual dredge lines in the field are concerned, the writer agrees with the general conclusions reached by Mr. Howard. However, although due regard must be given to all costs (such as running costs, amortization, etc.), the most important guides in designing new dredges and dredge lines are the results obtained on pipes in the laboratory, transporting material similar to that to be used in the field.

The economical velocity is practically the aforementioned critical velocity, and this must be obtained with as high a concentration as feasible. The minimum energy per unit of sand volume, obtained for each concentration when the mixture is flowing with a critical velocity, diminishes when concentration increases. This energy would reach a true minimum for high concentrations which would not be feasible with the technical equipment now available.

Common experience with dredge lines has shown that the most serious difficulty in running dredging equipment arises from the danger of sudden plugging of the line when pumping in the partial deposition phase. Such plugging is likely to occur not only in long lines pumping coarse material, but also in any length of pipe or size of material. This danger of plugging causes most dredge lines to be run at velocities that are far from economical; and until now the so-called economical velocity has been only of academic interest when the cost of re-starting flow in a plugged pipe was taken into account.

The fact that, with proper design, the pump can be given special characteristics that avoid the possibility of jerking and plugging is, then, of prime interest. Consider one of the older dredges in Nantes harbor. It has a main engine of 1 000 hp with a centrifugal pump of a type that could pump Loire River sand only for a distance of approximately 900 m to 1 000 m (2 950 to 3 300 ft), using a 580 mm (23 in.) pipe. At this distance, the sand volume pumped was limited to about 575 cu m (750 cu yd) per hr, under conditions in which sudden plugging was a real menace. The entire design of this dredge was made according to what is considered good normal practice.

In the new pumps, with the same engine and power consumption, the pumping distance is now 1 800 m (5 900 ft) with the same pipe diameter. At this distance, the sand volume pumped is normally 1 100 cu m (1 440 cu yd) per hr, with the same grade of sand as previously used. The danger of sudden plugging is now practically eliminated.

Since the concentration of the mixture thus pumped was more than 25% (bulk volume), the efficiency of the pump was about 80 per cent. There was much less wear in these pumps than in the older ones.

In the same general research, the suction heads have been perfected so that higher concentrations can be pumped. This is of major interest when dealing with fine sand, which can then be pumped away easily. Such fine sand, especially when mixed with muddy silt, is not so easily picked up and carried in suspension because natural deposits are often quite compact. A special suction head was designed using the entire flow of water to disintegrate the soil, the energy being supplied by the pump itself. A great uniformity is obtained without the risk of plugging. Concentrations are greater than 20% when working in compacted sand of very high permeability. The best dimension for the suction pipe has been derived for each condition.

The foregoing shows conclusively that the study of sand transportation in dredge pipes is most effective when theoretical knowledge of hydrodynamics and hydraulics is combined with experiments on small size test pipes. Although accurate measurements of the performance of big dredge lines in the field are difficult to obtain, such observations of field behavior and the interpretation of phenomena observed are essential.

The transportation of sand in pipes is a complex problem in the partial deposition phase. The flow condition depends largely for its stability on the characteristic of the kinetic head; and, although the flow conditions might be as described by Mr. Howard in certain pumps, the writer feels that, with a proper design, the flow condition could be much improved, and thus a genuine

economical velocity could be used. As both the pump and the suction head must be able to produce the high concentration that the pump now is able to drive through the line, it seems difficult to study the behavior of the line alone.

Although the laws governing all these phenomena appear to be extremely complex, the results noted thus far are gratifying. The difficulty of the task is no reason for abandoning research. Even if the final goal is still far removed, it must be reached, step by step, and investigators are still working in a stage where great strides can be made.

A broad exchange of ideas between workers in this field will foster progress and Mr. Howard deserves credit for making his results available to the Engineering Profession.

Acknowledgment.—The researches conducted at Nantes, mentioned in this discussion, were formerly conducted by Mr. Siegfried, under Chief Engineer Notte, and later by the writer. Technical advice came from Messrs. Louis Bergeron and Paul Bergeron, who designed the pumps, and also supplied valuable technical advice.

PIERRE F. DANEL,³² Assoc. M. Am. Soc. C. E. (by letter).^{32a}—Knowledge of turbulent flow has been much advanced in recent years, both through theory and experiment. Researches on sediment transportation in open channels that have been reported show encouraging results. Laws governing sediment transportation in pipes have much in common with those of open channels,

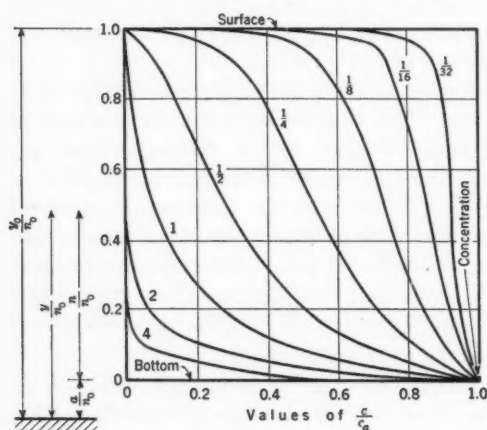


FIG. 14

and, as long as the basic differences are understood, much can be derived from one field to benefit the other. Among the most important differences is the effect that the free surface in open channels has in damping turbulence. This effect will create a very different distribution of time, average velocities, turbulence intensities, and sediment concentrations.

Fig. 14 shows some distribution curves of different typical sediment concentrations, arranged according to their relative state of turbulence.³³ The

effect of the free surface damping out turbulence is apparent as the concentrations diminish rapidly near the free surface.

Fig. 15 shows the approximate concentration distributions for a closed pipe. The bottom part of the curves is similar to that of open channel but the top is

³² Research Engr., Neyret-Beylier Piccard Pictet; Director of Hydr. Laboratory, Ecole des Ingenieurs Hydrauliciens, Grenoble Univ., Grenoble, France.

^{32a} Received by the Secretary December 14, 1938.

³³ "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, Assoc. M. Am. Soc. C. E., McGraw-Hill Book Co., New York, N. Y., 1938, p. 342.

quite different. When all the sediment is in suspension in the pipe, and if the concentration distribution is fairly uniform (see Curve A or Curve B, Fig. 15), the loss of head in the depth of the mixture is the same as that of water (as reported by Miss Blatch,² Mr. Gregory,³⁴ and at Nantes, France). In this case, the conditions are nearly those of an homogeneous fluid.

When the mean velocity is decreased the concentration distribution will no longer be uniform and, at the same time, when deposition occurs, the concentration distribution curve is of a type similar to Curve C or Curve D, Fig. 15. When the maximum size of the particles is small enough, a curious phenomenon may occur, as it did in experiments reported by Mr. Gregory. The loss of head (in depth of mixture) in a limited range of velocities may be less than that of clear water. This may be explained as being due to the damping effect of density gradation on turbulence, just as the density variations with height cause that well-known calmness of the atmosphere at sunset. The atmosphere is fairly homogeneous, however, whereas the mixture is not. If the particles are coarse their continued falling through the fluid creates a turbulence of its own, and there is little likelihood that the density variation can reduce turbulence to the point where the loss of head is less than that of water. When, on the contrary, the maximum size is such that the free fall of the grains would occur in laminar conditions (that is, particles of less than 0.1 mm), it is quite possible to get a loss of head, less than that of clear water, in a very limited range.

In the experiments conducted by Mr.

Howard, the grain sizes are too large to permit such a phenomenon to occur, but that must not be taken as a general proof that its occurrence is impossible.

In an open channel, the influence of a sudden deposition somewhere cannot be transmitted faster than surface waves, and the pressure changes involved are always small, whereas in closed pipes the pressure changes are transmitted at the velocity of sound in the pipe which is many times that of the speed of surface waves, and the pressure changes involved may be fairly great.

The jerky motion described by Mr. Howard, and the sudden plugging of the dredge pipe, do not have an exact parallel in open flumes, which shows that the comparison must not be extended too far. However, in mountain streams, mud flows of very high concentrations are stopped suddenly, sometimes in a manner that reminds one of the plugging of dredge pipes. In open channels only gravity flow is considered, whereas in closed pipe the kinetic head is generally produced by a pump, and flow conditions may vary considerably according to the pump characteristics, as accidental disturbances may or may not be smoothed out.

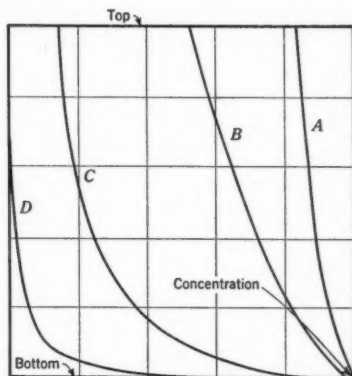


FIG. 15

² Transactions, Am. Soc. C. E., Vol. LVII (1906), p. 307.

³⁴ Mechanical Engineering, June, 1927, p. 609.

The design of pumps giving a stable performance outside of the cavitation range is a great step toward economy in the transportation of sand in pipes because, not only can a greater quantity be pumped, but the wear is much less.

In connection with the construction of dams the writer has been engaged for sometime in laboratory tests by throwing stones in flowing water. The theory of S. Isbach³⁵ has been checked and its scope has been broadened considerably. The almost perfect check between theory and experiment is very encouraging and, for material that is sufficiently coarse, the possibility of solving the deposition problem of gravel transportation in dredge pipe is not remote.

³⁵ "The Construction of Rockfill Dams by Dumping Stones in Running Water," S. Isbach, Leningrad, 1932. Translated by A. Dovjikov, U. S. Engr. Office, Eastport, Me., September, 1935.

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DISCUSSIONS

PRINCIPLES APPLYING TO HIGHWAY ROAD-BEDS

Discussion

BY MESSRS. A. F. GREAVES-WALKER, H. Z. SCHOFIELD, W. H.
CAMPEN, BERNARD E. GRAY, W. J. TURNBULL,
AND BERT MYERS

A. F. GREAVES-WALKER,²¹ Esq. (by letter).^{21a}—An excellent argument for the necessity of a better knowledge of ceramic materials and ceramic fundamentals on the part of highway engineers is presented in the paper by Mr. Mullis, which describes the properties of the ceramic materials used in highway construction very well. A study of these properties should be helpful to all highway engineers, and particularly to those who are confronted with the problem of constructing long-lived secondary highways. *

There is no question about the structural properties of earth masses being enhanced and made more highly resistant to water penetration by increasing their density. Considerable research work has been done by ceramic engineers in an effort to produce maximum density in both plastic and non-plastic bodies. In plastic bodies (clay), it has been found that the addition of acids or bases, depending on the pH-value of the clay, will increase both the plasticity of the mass and the dry density. Common or waste acids, or such bases as lime, calcium chloride, salt cake, soda ash, etc., may be used, but care must be taken to add the correct quantities. This depends on the pH-value of the material. Under any circumstances only small quantities are required to give desired results.

Much research has also been done on particle packing, and from the results it is possible to calculate beforehand the percentages of the coarse, medium, and fine fractions necessary to obtain the maximum density with any particular material or materials. Clays vary widely in their mineralogical composition and especially in their content of "clay substance," the colloidal hydrous alum-

NOTE.—The paper by Ira B. Mullis, Esq., was published in September, 1938, *Proceedings*. This discussion is printed in *Proceedings*, in order that the views expressed may be brought before all members for further discussion of the paper.

²¹ Pres., The Inst. of Ceramic Engrs.; Prof., Ceramic Eng., Univ. of North Carolina, Raleigh, N. C.

^{21a} Received by the Secretary October 17, 1938.

inum silicate which gives to clay its peculiar properties. The content of "clay substances" controls the bonding power. Many clays which to the "feel" are extremely plastic have low bonding power, due to a low content of "clay substance." When used as binders such clays produce "open" or porous bodies when dry, and thus readily absorb water.

Pressure is very important in producing dense bodies when either clay-bonded or non-plastic masses are considered. With increased pressure the water of plasticity can be reduced increasingly which, in turn, reduces shrinkage where clay bonds are used.

The effect of pressure on density varies considerably with the materials. Non-plastics like crushed stone which have hackly fractures, offer great resistance to movement under pressure, whereas smooth materials like gravel offer much less. Consequently, under equal pressure, the gravel will form the denser mass provided both are composed of fractions of the same size.

Mr. Mullis' investigations are unquestionably along lines that will result in greatly improved road-beds and fills. In the manufacture of a good ceramic product and the making of a good road-bed there is much common ground.

H. Z. SCHOFIELD,²² Esq. (by letter).^{22a}—It is of much interest to note that the conclusions (notably Conclusions (1) and (2)) of this paper are verified by results of researches on the effect of de-airing the earth materials used in the stiff-mud manufacture of structural ceramic products.

By stiff-mud extrusion from a laboratory brick machine, Messrs. J. O. Everhart, C. R. Austin and W. C. Rueckel²³ prepared specimens of three shales, a glacial surface clay, an alluvial surface clay, three coal-formation clays, a stoneware body, and a table-ware body. Duplicate specimens were then prepared in which the materials, immediately before extrusion from the brick machine, were de-aired by direct evacuation of air from the chamber of the brick machine. The effect of de-airing (and the consequent reduction in porosity and increase in density) on the physical properties was then determined. De-airing increased plastic strength both in tension and compression; it increased the dry transverse strength and modulus of elasticity; and, in all but one case, it caused a marked increase in the water-slaking time of the dry specimens.

W. H. CAMPEN,²⁴ Assoc. M. Am. Soc. C. E. (by letter).^{24a}—Much care has gone into the preparation of this paper. Mr. Mullis should be especially commended: (a) For his analysis of the steps involved in densifying water-soil mixtures; (b) for the classification for road construction of a number of typical oil deposits; (c) for clearly showing the effect of water content on the structural properties of soils; and (d) for the application of the information in the design of road-beds.

He assumes that a maximum pressure of 100 lb per sq in. is applied to surfaces on highways and then suggests that the entire upper 2 ft of the road-bed

²² Research Director, National Paving Brick Assoc., Washington, D. C.

^{22a} Received by the Secretary November 12, 1938.

²³ *Bulletin No. 74*, Ohio State Univ. Eng. Experiment Station, November, 1932.

²⁴ Pres. and Mgr., Omaha Testing Laboratories, Omaha, Nebr.

^{24a} Received by the Secretary November 14, 1938.

be constructed to support this load. There is no doubt in the writer's mind that, if all the soil or soil-mixture courses within this thickness are compacted so as to restrict their water capacity to 70% of their plastic limit, the resistance required will be developed. However the pore space called for is lower, and the thickness is higher, than is necessary for satisfactory road-beds. Some tests made in connection with the construction of the runways of the Omaha (Nebr.) Airport and other roadways will support the writer's contentions.

Assume that a road is required to carry, safely, and at all times of the year, a vehicle having 10 000-lb wheel loads and 100 lb per sq in. of air pressure in its tires. Assume, furthermore, that a soil is available which will pass a No. 40 sieve (with a plastic limit of 20); and a soil-gravel mixture containing material 27% smaller than a No. 40 sieve (plastic limit, 20). The relation between the load-bearing value and the water content, in dry weight per cubic foot, or pore space, is shown for these materials in Table 2. Column (3) is obtained by dividing the percentage of water used by the water content of the sample at plastic limit, and multiplying by 100. The plastic limit is determined on the part of the mixture that passes a No. 40 sieve.

TABLE 2.—RELATION BETWEEN LOAD-BEARING VALUE AND WATER CONTENT

WATER CONTENT EXPRESSED AS A:			AGGREGATE		Percentage of air voids	Load-bearing value, in pounds per square inch
Percentage of the Dry Weight		Percentage of the plastic limit	Weight per cubic foot, dry	Percentage by volume		
By weight	By volume					
(1)	(2)	(3)	(4)	(5)	(6)	(7)
(a) SOIL (SPECIFIC GRAVITY, 2.62)						
25.5	39.3	127	96.2	59.0	1.7	13
24.5	38.8	122	98.6	60.5	0.7	18
22.7	37.2	113	102.5	62.9	0.0	30
20.3	35.7	101	105.6	64.8	0.0	41
19.6	33.8	98	107.4	65.9	0.3	55
18.3	32.3	91	110.0	67.5	0.2	100
17.0	30.1	85	111.0	68.1	1.8	200
16.5	28.9	82	109.1	67.0	4.1
(b) SAND-GRAVEL BINDER MIXTURE (SPECIFIC GRAVITY, 2.61)						
10.4	21.3	180	127.0	77.8	0.9	13
8.1	17.2	137	132.3	81.2	1.6	42
7.4	16.1	124	134.7	82.7	1.2	200
6.4	13.7	106	133.5	81.9	4.5	320

The mixture is compacted in a mould 8 in. in diameter and 4 in. deep. Without removing the base of the mould a round bearing plate, 2 in. in diameter, is placed in the center and loaded until a $\frac{1}{8}$ -in. penetration is produced. The load, supported for 5 min at this penetration, is converted to pounds per square inch (see Column (7), Table 2). Readings can be taken at other penetrations but $\frac{1}{8}$ -in. is taken as representative of surface failure.

As shown by Mr. Mullis, the load-bearing value (he calls it the resistance to pressure) increases as the density increases, or as the pore space decreases,

it being understood that the pore space is nearly filled with water. It is to be noted also that at maximum density the load-bearing value is 200 lb per sq in. for both materials, and that the water content is 85% of the plastic limit in the case of the soil (Column (3), Table 2), and 124% in the case of the soil-gravel mixture (allowance is made for 1% absorption in the coarse aggregate). Since a resistance of 200 lb per sq in. satisfies Mr. Mullis' requirements, it would be wasteful to compact with less water and to a higher density. It might be added that the results obtained with the two materials given are representative of mixtures in general within the writer's experience. These tests have been made on soils having plastic limits of from 15 to 33, and on soil-coarse aggregate mixtures having plastic limits of 10 to 30 and maximum sizes as great as 2 in.

As to the thicknesses required: Since it is good common practice to use a coarse aggregate mixture, covered with a bituminous mat, on the upper part of the road-bed, assume that the upper part of the road will be composed of 2 in. of mat and 6 in. of a soil-gravel mixture. The mat will reduce the 100-lb load by distribution so that only 57 lb will be delivered to the base. The base (Table 2(b)), packed to maximum laboratory density, can safely handle this load, and, in turn, will deliver a load of about 17.5 lb per sq in. to the sub-grade. For adequate protection the sub-grade should be constructed to support twice this load, or 35 lb. The soil shown in Table 2(a) will do this job well if compacted with a water content equivalent to 101% of its plastic limit, or to 95% of laboratory density.

The next question is: How thick shall the compacted sub-grade be made? The full 16 in. demanded by Mr. Mullis would deliver a load of about 7 lb to the 24-in. level (it is assumed that the compacted soils have a load distribution ability equal to 50% of the coarse aggregate mixture). There are many locations where this type of construction is needed because of the water content of the natural sub-grade. There are other locations (and they are much greater in number) where the soils have pore spaces of about 50%, but where water can be prevented from reaching them. At these locations the water content is such that high load support prevails. In these cases, then, advantage should be taken of the high sub-grade support and the construction should be limited to compacting the upper 6 in. to 90% of laboratory density. As a matter of fact, many heavily traveled highways are built on this principle and would fail if enough water could reach the sub-grade to fill the pore space.

Mr. Mullis' requirement of not more than 40% of pore space in materials below the 2-ft level is reasonable except when applied to certain types of clay; but, as has been stated herein, this should only be considered where water might be accessible. In connection with this condition, it must be borne in mind that the load-bearing value at any given water content varies with the plasticity index. For instance, a clay that has a plasticity index of 60 has a load-bearing value of 100 lb per sq in. at a 26% water content, whereas a soil having a plasticity index of 30 has only a 25 lb per sq in. load-bearing value at the same water content.

The construction and successful performance of runways at the Omaha Airport will serve as an example of the writer's contentions. A typical cross-section shows 1-in. sheet asphalt, 2-in. asphaltic concrete binder, 6-in. soil-

gravel mixture, and 6-in. compacted soil. The base was compacted to a maximum density and the sub-base (the upper 6 in. of the sub-grade) was compacted to at least 95% of maximum density. The undisturbed sub-grade is saturated with water and has a maximum pore space of 50 per cent. A modern airplane having a wheel load of 13 500 lb and 51 lb of pressure in its tires has operated for two years on parts of the runways without any signs of failure.

BERNARD E. GRAY,²⁵ Esq. (by letter).²⁶—It is indeed encouraging to find attention being given in larger measure to the problems of earth foundations, because through application of even the broader principles governing soil densities, great saving in construction costs, particularly of highways, may be obtained. Mr. Mullis has outlined these principles in a concise and logical manner.

It is unfortunate indeed that so many engineers have only a slight knowledge concerning soil behavior and are little concerned about it. This attitude arises from a fundamental misconception in regard to the place of soil in the construction "picture." To many, "dirt is dirt," and their designs are usually made massive to take care of the worst anticipated conditions, with no thought or provision for possible modification of the soil itself. To check the truth of this statement, it is only necessary to watch some heavy grading job, and to note the blithe manner in which the weathered and more stable soil is placed at the bottom of a fill, and the raw, unstable soil on the top. The writer assumes no virtue in this regard, having built many, many miles of highway over past years in just the same wrong manner, but, in the light of present knowledge, such practice should be modified, and the valuable stable soils should be saved and utilized in the sub-grade where they belong. When engineers begin to think of relative values in soils as they do for the various manufactured structural materials, then they will be truly on the way to making some genuine savings in foundation designs.

Shear necessity is the most compelling motive to action, and about 1918 the writer, in making a study of old survey notebooks covering highway locations in Virginia and West Virginia, was impressed with the comments therein respecting the character of the ground traversed. The party chief had carefully noted his proposed location with especial reference to soils and their behavior under temperature and moisture changes. Some of these soils had remarkable stability, as evidenced by their solid condition under wagon traffic, so much so that when the writer later built modern type roads, largely on these old locations, he experimented by placing variable thicknesses of macadam over the most stable soils. In one particularly good section, only 3 in. of limestone macadam was built using the compacted soil as a base, except for a thin layer of screenings and dust placed over the soil as a leveling course. This section is still (1938) under traffic and its behavior has been better than some of the 8-in. and 10-in. adjacent sections having sub-grades of less bearing power.

²⁵ Chf. Engr., The Asphalt Inst., New York, N. Y.

²⁶ Received by the Secretary November 14, 1938.

On some of the secondary roads, bituminous surface treatments were placed directly on these soils and served traffic adequately for a number of years before additional thickness was required to distribute the increasing wheel loads better. Taking advantage of these favorable conditions it was possible to improve the system as a whole many years sooner than would have been the case had conventional designs only been used.

There is little to criticize in Mr. Mullis' paper, but perhaps some emphasis on certain paragraphs may be helpful, as follows: Under the heading, "Loose, Amorphous, Plate-Like, Fibrous, and Cellular Matter," he states: "Loose material, regardless of other attendant structural properties, should have no place in a road-bed." Nothing could be more true, and yet it is continually disregarded. Many bases have failed, not because of poor quality, but because they were not consolidated, and the surface placed thereon became distorted later as further compaction occurred under traffic. It is common practice to write specifications as to thickness of such courses, and quite uncommon to have any standard as to density of the completed work. Without such density control, thickness is a meaningless measurement. Time and again, thicknesses have been increased, when all that was required was to obtain uniform density of the thinner layer. This conclusion is borne out in the statement by Mr. Mullis in the text following Equation (7):

"Many bituminous mat-road surfaces which had failed due to the apparent weakness of the road-bed have been examined. Of these failures no case is recalled in which the bulk density of the road-bed was in excess of 1.50. * * * the theoretical water capacity, therefore, is of 28.9 per cent."

Regardless of pavement type, it should be recognized that stabilized road-beds are required for long life and low cost of repairs.

Under the heading, "Brittleness, Elasticity, and Plasticity," it is stated that: "Loose masses (low density) under stress manifest no elastic properties, but solidified masses (high density) show elastic properties as do other solids under stress."

The rational design of the so-called "flexible type" pavements is predicated upon this axiom. It applies both to the road-bed and the pavement. When road-beds (sub-grades) are designed for a given minimum density under the most adverse conditions, it is then a simple matter to calculate the depth of pavement required to distribute wheel loads so that its bearing capacity is not exceeded. In respect to wheel loads, the comment under "Design of Road-Beds" should be noted. Balloon tires have modified pavement design. Present wheel loads are so distributed over a large contact area that there is little, if any, more destructive action under the heaviest vehicle than was formerly experienced under a two-horse wagon. As Mr. Mullis states, present tire pressures are rarely in excess of 75 lb with the trend toward less pressures. Thus, a design load of 200 lb per sq in. gives an ample margin of safety.

Under the heading, "Design of Road-Beds: Road-Bed Protection," it is stated that:

"The agency most destructive to compacted road-beds is weather. * * * A pavement or bituminous surface alone is not sufficient protection against the

weather. Pavement cracks generally develop and joints are seldom watertight. For this reason provision should be made to prevent excessive moisture changes near the sub-grade and in the shoulders."

This is a most pertinent comment, and should have much influence on design. The increased use of bituminized shoulders and sub-grades is recognition of the need for preventing water from softening sub-grade or shoulder support. A striking example of the merit of such protection is to be found in Connecticut, where during the flood of September, 1938, practically no damage occurred to pavements where such shoulder protection was used.

Although the inherent values of designed sub-grades are not yet widely appreciated, a few engineers have begun to build roads by such means. Some question has arisen in respect to the rational design of flexible pavements making use of variable sub-grade support. Valuable contributions on this subject have been made by W. S. Housel,²⁶ Assoc. M. Am. Soc. C. E., W. S. Downs,²⁷ A. T. Goldbeck,²⁸ and G. E. Hawthorn,²⁹ Members, Am. Soc. C. E., and many others. Although there may be some difference of opinion as to the manner of stress distribution in soils, the conventional distribution for an elastic solid is certainly conservative for designed bituminous mixtures of known stability. Accordingly, the writer suggests a formula to control design thickness that gives results which, in the light of road behavior, are more than adequate for modern traffic.

In the usual equation, the pavement thickness is calculated by assuming that the load is applied at a point, whereas actually it is applied over a considerable area depending upon the character of the vehicle and its tire equipment. Although this area varies with different loadings and different pressures (in pneumatic tires), Table 3 shows the average conditions in round numbers.

TABLE 3.—AREA OF TIRES IN CONTACT WITH ROAD

Description	UNIT WHEEL LOAD, IN THOUSANDS OF POUNDS:				
	2	4	6	8	10
Area of tire or tires in contact with road, in square inches.....	25	57	75	95	112
Radius of equivalent circle, in inches.....	2.8	4.2	4.9	5.5	6.0

L. W. Teller and J. E. Buchanan,³⁰ Assoc. Members, Am. Soc. C. E., indicate that maximum load intensities occur in two transversely symmetrical zones rather than at the geometric centers of the areas, and the differences in load per square inch are not as great as was formerly supposed.

²⁶ "Design of Flexible Surfaces," by W. S. Housel, 23d Annual Highway Conference, Univ. of Michigan, February, 1937.

²⁷ "The Cost of Providing Highways Suitable for Various Classes of Vehicles," by W. S. Downs, *Special Bulletin*, Univ. of West Virginia, September, 1933.

²⁸ "Researches on the Structural Design of Highways by the United States Bureau of Public Roads," by A. T. Goldbeck, *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 264.

²⁹ "A Method of Designing Non-Rigid Highway Surfaces," by G. E. Hawthorn, *Bulletin* 83, Univ. of Washington, August, 1935.

³⁰ "Experimental Determinations of the Variations in Pressure Intensity Over the Contact Area of Tires," Highway Research Board, December, 1937.

It is apparent, therefore, that in place of a cone of distribution, there is actually the frustum of cone, and that the true radius of the area of load distribution over the base or sub-grade is equal to the thickness of the pavement plus the radius of tire-area contact.

In Fig. 5, let W = wheel load, in pounds; s = safe bearing value of the sub-grade, in pounds per square inch; t = required thickness of pavement, in inches; r = radius of cone; and e = the dimension shown.

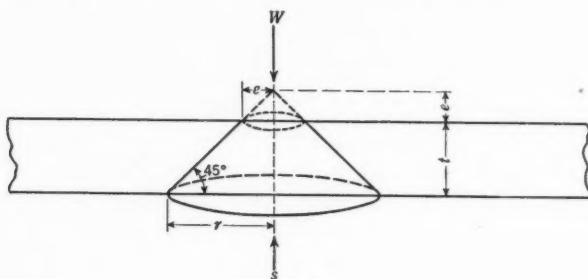


FIG. 5

Then,

$$W = \pi r^2 s \dots \dots \dots (8)$$

but, $r = t + e$; and $W = \pi (t + e)^2 s$; that is,

$$t = 0.564 \sqrt{\frac{W}{s}} - e \dots \dots \dots (9)$$

The bearing capacity is composed of two separate factors of strength, one which is dependent upon the perimeter of the area, called perimeter shear, the other dependent upon the area itself, termed the strength of the pressure bulb. This latter factor often has been the only one given consideration in determining the thickness of flexible pavements, and yet in bearing areas of small size, the factor of perimeter shear, in giving support value to the sub-grade, may be the larger of the two. Professor Housel,²⁶ who has made a number of studies in this regard, gives the following equation for these relationships:

$$W = A S = m P_A + n A \dots \dots \dots (10)$$

in which W = total load carried on a given area, in pounds; A = area, in square feet; P_A = perimeter of area, in feet; m = perimeter shear, in pounds per linear foot; n = developed pressure, in pounds per square foot; and,

S = bearing capacity, in pounds per square foot $= m \frac{P_A}{A} + n$.

In 1932, Roland Vokac,³¹ Assoc. M. Am. Soc. C. E., then with the Michigan State Highway Laboratory, reported some very informative studies in connection with the design of low-cost bituminous mixtures. Making use of Equation (10) (expressed in inches instead of feet to conform to the size of

³¹ "Studies in the Proportioning of Low Cost Bituminous Mixtures of Dense Graded Aggregate Type," Tenth Annual Asphalt Paving Conference, 1932.

suitable laboratory samples) he found that the values of m and n could be determined for any mixture through the use of a simple laboratory machine. Numerous mixtures of varying behavior under traffic were tested and the laboratory results consistently confirmed the road behavior, not only relatively, but in accurate fundamental terms of bearing capacity. Also, contrary to the general assumption that the highest concentration of pressure always occurs under the center of the loaded area, it was found that up to certain loads there is actually a negative value for n because the perimeter shear value, m , is so high that all the load is carried at the edge of the bearing area. As the load further increases this shear value builds up first, followed later by the increase of pressure resistance until both become constant, at which point the ultimate strength of the material is reached.

Professor Housel also shows that,²⁶ "The importance of the time element in interpreting the results of test on all varieties of soils cannot be over-emphasized." By this is meant the difference in the effect of load distribution between a load suddenly applied and then immediately removed, and one applied gradually over a longer period of time. An interesting example of this difference is familiar to all in the incident of school days in sliding across thin ice without difficulty where it would have been impossible to have walked across without breaking through. Still another factor contributing to the supporting power of a sub-grade is the effect of surcharge due to the weight of a superimposed layer of well bonded material. This is particularly marked in sub-grades which depend principally upon internal friction for stability and in which there is little cohesion, as in the case of loose sand.

Although this discussion could be prolonged indefinitely without exhausting its potential values, it is hoped that some additional attention may have been focused upon the paper of Mr. Mullis. It is believed that the following conclusions can be derived from reading it:

(1) Although various types of road surfaces have been used over a long period of time, only in the past few years has there been any effort to appraise conditions under which they should be selected and utilized in relation to traffic needs.

(2) The most important part of the road structure (the natural soil foundation) has been almost completely ignored in making an engineering analysis. Even now only a few States follow any definite procedure in this regard.

(3) There are two methods of designing a road surface: (a) To make it so thick that it stands in spite of a bad foundation; and (b) to place a wearing course exactly thick enough to supplement the sub-grade support. The first method often entails unnecessary expense.

(4) There is always a way to utilize local materials for road construction; it only requires intelligent engineering study to develop the requisite technique. This is particularly true in respect to sub-grade stabilization.

(5) The load supporting capacity of a sub-grade depends upon two characteristics: Internal friction and cohesion between the particles making up the soil. These vary according to the moisture content (especially the latter), and beyond a critical point result in reduced capacity to carry load.

(6) Control of moisture content, then, is of the greatest importance and yet engineering attention to this matter is often more a matter of talk than of action.

(7) Having controlled moisture through appropriate methods according to job conditions, it is then required to determine the thickness of surfacing for traffic. This involves two studies, one as to the minimum support to be derived from the sub-grade, either natural or as modified, and the other as to the capability of the selected surface, to transmit load without distortion. Soil classification has proceeded to the point where a relatively few general groups cover all locations, so that all that is required is to apply methods for measurement already established in other fields. There are several laboratory machines for determining the stability of wearing course mixtures. There only needs to be a greater use made of these methods and facilities.

(8) To utilize inherent soil values to best advantage, work should be done in a uniform manner. By proper attention to grading operations the finished sub-grade can be made of uniform load-supporting character, with resulting uniformity in wearing course. It is less expensive to prepare a high support sub-grade with a relatively thin surface course to meet a given traffic load, than to build a thick surface on a "hit or miss" sub-grade. What seem to be low prices for unclassified excavation in road-grading operations are often actually very costly because good soils are frequently wasted.

(9) According to well-known laws it is possible to use a formula for the determination of the necessary thickness of wearing course to meet any condition. That this formula is not the same as the one used in designing rigid pavements is simply a recognition of the fact that there are basic differences between elastic and plastic substances.

W. J. TURNBULL,³² ASSOC. M. AM. SOC. C. E. (by letter).^{32a}—In this paper, the author has presented a clear picture of the functions of a soil in a road-bed. He has reviewed the various states of matter and the way in which each one influences the action of a soil in the road-bed, when placed at varying degrees of compaction. The definitions of the various characteristics of a soil, as well as the various soil types, are believed to be especially good.

The gist of the paper, as interpreted by the writer, is to the effect that increasing the soil density: (1) Enhances the structural properties of the soil; (2) increases its resistance to water penetration, thus acting to decrease weathering ravages; and, generally, (3) improves the desirability of the soil in all respects as the principal component in a road-bed. In general, this is true. However, some soils are known to have more detrimental swelling properties at the higher densities, and, consequently, this feature should not be neglected in the study of a soil to be used in a road-bed.

Mr. Mullis introduces the term "yield point due to water content" as marking the end of the elastic state and the beginning of the plastic state. He also states that it is approximately equal to Atterberg's lower plastic

³² Soils Engr. and Chf. of Laboratory, The Central Nebraska Public Power and Irrig. Dist., Ogallala, Nebr.

^{32a} Received by the Secretary November 17, 1938.

limit in clays, and to 75% of Atterberg's liquid limit in silts and fine, friable materials. Further, the inference is made that the plastic state is undesirable. The writer feels that the early stages of the plastic state are not undesirable provided the strain produced in the road-bed on the application of the maximum wheel load is not beyond an amount which is harmful to the surface-protection mat. This is analogous to the fact that a structure may withstand successfully 2 in. of settlement, whereas 4 in. would be very harmful. The early stages of the plastic state may even be more desirable than the early stages of the elastic state, due to the fact that the latter borders near brittleness; whereas the strain in the former state produced by the maximum wheel load is not detrimental to the wearing surface. As stated by the author, a brittle (dry) road-bed is undesirable unless the soil possesses high tensile strength, which very seldom if ever occurs. Furthermore, the elasticity of a soil increases with increasing density, other factors remaining equal. The preceding statements have been made on the assumption that a soil placed at a given density passes through the brittle state into the elastic state and into the plastic state with increasing moisture content.

In the design of the road-bed, the author indicates the "factor of safety against water absorption" as the ratio of the "yield point due to water content" (lower plastic limit) to the actual water capacity of the soil. He further indicates that the top 2 ft of a road-bed should have a factor of safety of 1.43 (70% of yield point), and that the pore space of all sub-grade soil below the 2-ft depth should not be greater than 40 per cent. Assuming a true specific gravity of 2.65, the dry weight of the soil below the 2-ft depth would be about 99 lb per cu ft. Assuming a true specific gravity of 2.65 and a lower plastic limit of 18% moisture, the dry weight of the soil with a factor of safety of 1.43 above the 2-ft depth would be about 124 lb per cu ft. It would be impractical, if not impossible, to obtain the latter weight by rolling, if the road-bed was to be constructed of a wind-blown silt loess material of uniform grain size. The only practical way to obtain this density would be by using suitable admixtures, which immediately brings the question of economy into the picture. The dry weight of 99 lb per cu ft is quite possible to obtain even in most poorly graded silts. It would seem to the writer that, in general, the factor of safety against water absorption as set by the author for the upper 2 ft may be somewhat high; however, for certain soils, it may be entirely practicable and satisfactory. It is impossible to lay down "hard-and-fast" rules that are applicable to all soil types. Certain general conditions undoubtedly apply to all soils. When used in a road-bed, however, each should be studied in detail and the various requirements modified to fit the individual soil.

The author is to be commended for presenting a definite method to follow in the design of road-beds. Such a method should tend to reduce the practice of "hit and miss" which has existed, all too often, in the past.

BERT MYERS,³³ Esq. (by letter).^{33a}—The highway engineer concerned with the economics of highway design will be interested in the limits suggested by

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^{33a} Received by the Secretary November 18, 1938.

TABLE 4.—CHARACTERISTICS OF SOIL TYPES

Item No.	Soil type	Position	ANALYSIS (PERCENTAGES)				DENSITY				ATTERBERG				PERCENTAGE OF WATER, BY WEIGHT, REQUIRED TO FILL PORES			Ratio road-bed moisture (lower plastic limit)	
			Gravel	Sand	Silt	Clay	Absolute	Proctor	Standard	Road-bed	Lower liquid limit	Lower plastic limit	70% of lower plastic limit	(14)	(15)	(16)	(17)		(18)
1	Clay	New cut	35	33	30	2.70	1.86	1.95	1.66	33.7	14.9	10.4	19.6	16.7	14.3	23.2	84.5	131.5	
2	Clay	New cut	33	31	30	2.73	1.72	2.00	1.69	35.7	17.5	12.3	20.4	21.5	13.5	22.7	90.1	116.6	
3	Clay (gumbotill)	New cut	0	5	35	60	2.74	...	2.13	1.44	64.0	22.0	15.4	23.9	...	10.3	82.6	108.6	
4	Clay	New cut	1	18	50	2.66	...	1.93	1.32	49.5	21.9	15.3	25.9	...	14.2	36.2	67.9	118.3	
5	Clay (shale)	Old road-bed cut	0	2	63	2.75	...	1.96	1.62	51.0	17.0	11.9	25.2	...	14.7	25.4	99.2	148.2	
6	Clay loam	New cut	1	35	39	2.57	...	1.66	1.16	52.0	27.6	19.3	31.3	...	21.3	47.3	66.1	113.4	
7	Clay loam	New cut	2	36	36	2.61	...	1.67	1.47	38.0	21.0	14.7	22.0	...	20.9	29.7	74.1	104.8	
8	Clay loam	Old road-bed cut	0	45	31	2.82	...	1.82	1.73	21.5	15.7	11.0	16.3	...	19.5	22.4	72.5	103.8	
9	Clay loam	Old road-bed cut	0	49	22	2.69	...	2.00	1.83	37.0	15.0	10.5	14.2	...	12.8	22.5	63.1	94.7	
10	Clay loam	New cut	2	41	28	2.67	1.81	1.96	1.68	37.0	16.7	11.7	20.5	13.6	17.8	22.1	92.7	122.8	
11	Loam	Borrow-pit	1	43	41	2.62	...	1.69	1.33	38.4	21.6	15.1	23.8	...	21.0	37.0	64.4	110.2	
12	Loam	Borrow-pit	0	47	36	2.64	1.82	1.68	1.37	27.6	16.2	11.3	19.3	17.1	21.6	35.1	54.9	119.1	
13	Loam	Old road-bed cut	1	44	38	2.68	...	1.69	1.44	25.9	16.7	11.7	19.3	...	21.9	32.1	60.1	115.6	
14	Loam	Road-bed fill	0	35	49	2.69	...	1.77	1.78	23.4	17.2	12.1	16.0	...	19.3	19.4	84.2	93.0	
15	Loam	New cut	1	40	44	2.69	...	1.75	1.67	23.6	15.1	10.6	12.5	...	20.0	22.7	55.1	82.8	
16	Silty clay	New cut	0	1	69	30	2.69	1.74	1.82	137	41.0	19.4	13.6	25.5	20.3	35.8	71.2	131.4	
17	Silty clay (muck)	Rejected borrow	0	7	52	41	2.35	...	0.99	98.0	74.0	51.8	101.6	...	58.4	102.3	90.3	137.4	
18	Silty clay (peat)	Rejected sub-grade	1	16	57	1.92	...	0.77	0.34	164.0	146.0	102.2	241.7	...	77.8	242.1	99.8	165.5	
19	Silty clay	Old road-bed cut	0	1	57	42	2.70*	...	1.85	1.51	51.0	20.0	14.0	26.0	...	17.0	29.2	89.0	130.0
20	Silty clay	Old road-bed cut	0	4	57	39	2.71	...	1.78	1.45	49.0	20.0	14.0	26.6	...	19.3	32.1	82.9	133.0
21	Silty clay loam	New cut (top-soil)	0	4	68	28	2.63	1.52	1.60	126	48.7	26.6	18.6	28.2	27.8	25.5	41.3	68.1	106.0
22	Silty clay loam	Old road-bed cut	0	1	71	28	2.68	...	1.77	1.63	36.0	19.0	13.3	19.9	...	19.2	24.0	82.7	104.7
23	Silty clay loam	Old road-bed cut	1	24	50	2.64	...	1.62	1.50	36.5	19.6	13.7	20.1	...	23.8	28.8	69.9	102.5	
24	Silty clay loam	Old road-bed cut	1	20	52	2.70	1.80	1.73	1.60	33.0	20.0	14.0	16.5	18.5	20.8	25.4	64.9	82.5	104.1
25	Silty clay loam	Road-bed fill	0	4	74	22	2.69	...	1.62	1.63	35.0	22.0	15.4	22.9	...	24.6	24.2	94.6	104.1

TABLE 4.—Continued

Item No.	Soil type	Position	ANALYSIS (PERCENTAGES)				DENSITY				ATTERBERG			PERCENTAGE OF WATER REQUIRED TO FILL PORES				Ratio road-bed moisture (lower plastic limit)
			Gravel	Sand	Silt	Clay	Absolute	Proctor	Standard	Road-bed	Lower liquid limit	Lower plastic limit	70% of lower plastic limit	Proctor	Standard	Road-bed	Percentage of road-bed pore space filled with water	
26	Silty loam	Rejected borrow	0	18	62	20	2.60	1.63	1.24	29.3	20.5	27.5	22.9	41.2	65.2	93.9
27	Silty loam	New cut	4	3	78	19	2.71	...	1.71	18.9	13.2	20.1	21.6	24.8	106.3	106.3
28	Silty loam	Old road-bed cut	4	29	53	14	2.70*	...	1.79	16.7	11.8	18.1	18.8	22.7	79.7	107.7
29	Silty loam	Old road-bed cut	0	2	78	20	2.70	1.80	1.72	29.6	13.7	20.1	...	18.5	21.1	36.5	95.8	178.1
30	Silty loam	Road-bed fill	1	30	52	17	2.71	...	1.73	17.7	12.4	19.9	20.9	29.7	67.0	112.4
31	Silt	New cut	0	1	81	18	2.70*	1.74	1.59	26.6	19.2	25.8	20.4	20.4	20.4	25.9	99.8	134.4
32	Silt	New cut	0	1	80	19	2.72	...	1.70	16.5	19.4	21.5	...	22.1	23.8	90.3	110.8	111.7
33	Silt	New cut	0	1	81	18	2.71	1.84	1.65	26.7	16.0	22.9	17.4	17.4	24.4	23.7	96.7	111.7
35	Sandy clay loam	Old road-bed cut	9	48	24	19	2.68	1.96	...	24.3	13.2	14.3	13.7	21.8	65.6	108.3
36	Sandy clay loam	Old road-bed cut	10	46	24	20	2.68	...	1.80	15.7	10.5	12.7	18.2	26.3	84.7	84.7
37	Sandy clay loam	Road-bed fill	3	51	23	23	2.69	...	1.97	16.9	12.4	8.7	14.2	...	13.6	22.0	64.5	114.5
38	Sandy clay loam	Old road-bed cut	7	47	26	20	2.69	...	1.85	18.2	16.0	11.2	15.3	...	16.9	17.7	86.1	95.6
39	Sandy clay loam	New road-bed back-fill	2	51	25	22	2.68	...	1.92	18.3	13.0	9.1	11.6	...	14.8	17.3	67.1	89.2
40	Sandy loam	New cut	9	53	20	18	2.69	...	2.02	17.6	21.9	11.8	8.3	...	12.3	19.7	62.1	103.4
41	Sandy loam	Borrow-pit	4	62	26	8	2.67	...	1.82	15.8	4.2	...	17.5	25.8	16.2	...
42	Sandy loam	Borrow-pit	0	53	34	13	2.66	1.85	1.72	15.3	22.5	13.9	9.7	16.5	20.5	27.8	24.2	48.2
43	Sandy loam	New cut	1	51	31	17	2.66	...	1.68	14.1	25.0	15.5	10.9	...	21.9	33.3	47.9	107.7
44	Sandy loam	Old road-bed cut	6	50	25	19	2.69	...	1.91	18.1	26.5	12.9	9.0	...	15.2	18.1	78.6	110.1
45	Sand	Borrow-pit	2	88	5	5	2.68	1.88	1.74	...	15.6	13.0	20.2
46	Gravelly sand	New cut	16	68	10	6	2.74	2.02	...	15.8	14.7	10.3	...	13.0
47	Sand	Old road-bed cut	8	79	8	5	2.75	...	1.75	1.61	20.8	22.0	71.4	106.8
48	Sand	Old road-bed cut	2	82	10	6	2.67	...	1.85	2.02	16.6	12.0	61.7
49	Sand	Old road-bed cut	0	87	9	4	2.66	1.80	...	1.85	18.0	...	16.5	78.2	...
50	Gravelly clay loam	Sub-grade back-fill	21	36	27	16	2.71*	2.00	1.97	17.7	36.0	17.0	11.9	12.6	13.5	13.7	64.3	74.1
51	Gravelly clay loam	Back-fill borrow	23	42	21	14	2.71*	2.07	1.99	...	31.0	15.0	10.5	...	11.4	13.4
52	Gravelly sandy loam	Back-fill borrow	35	48	9	8	2.68*	2.16	2.06	...	23.6	14.1	9.9	...	9.0	11.2
53	Gravelly clay loam	Sub-grade back-fill	32	26	20	2	2.70*	2.01	1.85	1.90	51.0	22.0	15.4	7.2	12.7	17.0	15.6	46.2
54	Gravelly sandy loam	Roll stone stabilized base	52	25	14	9	2.66*	2.12	...	2.20	22.7	13.3	9.3	6.6	9.6	...	8.0	82.5
55	Gravelly sandy loam	Soil aggregate stabilized base	31	54	10	5	2.66*	2.17	...	2.26	18.8	12.3	8.6	8.9	...	6.6	72.7	39.0

* Estimated.

Mr. Mullis for the physical properties of a satisfactory road-bed, and the cost of building a structure that will meet these requirements. He will also be interested in learning whether soil to which these properties have been imparted will continue to possess these properties after being exposed to varying conditions of temperature, stress, and opportunity for the absorption of moisture.

It is suggested that the road-bed should have an elastic limit of not less than 200 lb per sq in. The method for determining this elastic limit is not described. If it is to be determined by subjecting a small sample of soil, such as might be cut from the road-bed, to compression without horizontal restraint, this value would seem to be high, and difficult and expensive to attain.

It is suggested that the pore space in parts of the road-bed more than 2 ft below the sub-grade should not exceed 40% of the volume. This limit seems entirely practicable and reasonable. Another suggestion is that the pore space in the upper 2 ft of the road-bed be reduced to such an extent that its theoretical water capacity is not greater than 70% of the lower plastic limit of the earth used.

The Bureau of Soils Surveys, Iowa State Highway Commission, has conducted some studies to determine the probable behavior of soils in such a condition under field conditions (see Table 4). Samples have been taken from stabilized gravel base courses and their density and moisture content determined. The results of these studies indicate that this material, which had a distribution of particle sizes and degree of compaction such that the theoretical water capacity was below the limit suggested by Mr. Mullis, did not become less dense or absorb moisture when exposed to the conditions existing on the road. Samples taken from bases that contained 8 to 10% of moisture at the time they were constructed, were found to contain from 3 to 7% moisture two years later. This was true even where the moisture content of the sub-grade soil, immediately beneath the base material, was as high as 24 per cent.

A laboratory study along this same line was conducted as follows: Samples of three different soils common in Iowa, and mixtures of these soils with pit-run gravel, were formed into cylindrical specimens 4.5 in. in diameter by 9 in. high. The specimens were compacted to the highest density that seemed practicable with present road-building equipment. The specimens were moulded in rubber tubes which covered the specimens throughout the test. The lower end of the specimen was held in the tube by a perforated metal disk and blotting paper. The specimens were frozen in a freezing chamber and then thawed in the laboratory air with their lower ends immersed in water. The freezing and thawing treatments were alternated ten times. The volume and weight of each specimen were determined after each alternation. The results of this series of tests are given in Table 5.

It will be noted that specimens in which the theoretical water capacity was greater than 70% of the lower plastic limit of the material (Column (10), Table 5) decreased in density and increased in moisture content under the test procedure. In these specimens the final moisture content was slightly greater than the lower plastic limit.

Specimens with theoretical water capacities of less than 70% of the lower plastic limit of the material did not decrease in density or gain materially in moisture content during the test. Specimens that did not decrease in density or increase in moisture content were composed of mixtures of soil and pit-run gravel.

TABLE 5.—RESULTS OF FREEZING AND THAWING ON THE DENSITY AND MOISTURE CONTENT OF SOIL SPECIMENS

Specimen	COMPOSITION (PERCENTAGES)		Percentage passing a No. 40 sieve	ORIGINAL CONDITION		CONDITION AFTER FREEZING AND THAWING		Plastic limit	Water capacity*
	Clay	Gravel		Density	Percentage of water	Density	Percentage of water		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
A	Loess		100	1.84	15.1	1.73	20.4	19.4	89
B	Loess		100	1.77	17.5	1.68	21.0	20.9	93
C	Glacial clay		77	2.01	10.7	2.00	12.2	12.1	100
D	30	70	40	2.16	8.1	2.18	8.0	12.8	67
E	10	90	26	2.20	7.4	2.20	7.5	13.0	60
F	30	70	41	2.10	9.8	2.10	10.0	15.2	67
G	13	87	30	2.17	8.3	2.18	8.3	14.1	60

* Percentage of plastic limit.

The values in Column (4), Table 5 would indicate that, in order to make these characteristic Iowa soils comply with the suggested requirement, they would need to be mixed with a sufficient quantity of coarser material to reduce the percentage passing the No. 40 sieve to about 40. In some localities, the cost of importing such a proportion of coarse material and mixing it with the soil locally available would be very great. This suggests that it might be cheaper to use the local materials to produce the best road-bed possible without the addition of coarser materials, and make up for the deficiency in the road-bed by constructing a stronger surface course.

Mr. Mullis has made a valuable contribution to the literature on the subject of the design of road-beds by presenting a paper in terms with which every engineer is already familiar.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

SETTLEMENT STUDIES OF STRUCTURES IN EGYPT

Discussion

BY MESSRS. L. C. WILCOXEN, TRENT R. DAMES,
AND EDWIN J. BEUGLER

L. C. WILCOXEN,⁹ Assoc. M. Am. Soc. C. E. (by letter).^{9a}—The last decade of research on the problem of the stresses and strains occurring beneath foundations has done much to consolidate the current theoretical knowledge of the subject. In the matter of large non-rigid foundations on certain types of soils, for instance, it is now quite generally recognized that differential settlements must occur.

Professor Tschebotareff's contribution is to put in the record important differential settlements of a number of structures built on clay and silt. His chief conclusion is to emphasize the necessity for settlement studies in all large cities. This conclusion deserves wide recognition.

It should be understood, however, that such knowledge will be of practical value only when methods for preventing such settlements have been developed. Two methods for doing this may be used: (1) By designing the structure with sufficient rigidity to sustain the stresses induced by the unbalanced footing loads; and, (2) by treating the foundation soils in a manner to offset their movements due to compression, consolidation, and plastic flow. The former has been consciously done in at least one case.¹⁰

The latter could undoubtedly be done by the principle of "mudjacking," now used in raising sunken highway pavements. It may be that at some future time "mudjacking" pipes will be extended deep into the foundation soil during construction and specifications will demand that, over a period of a year or two, the contractor restrict the building settlements to certain limits.

NOTE.—This paper by Gregory P. Tschebotareff, M. Am. Soc. C. E., was published in October, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

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^{9a} Received by the Secretary November 16, 1938.

¹⁰ "Economics of Rigid Frame for Building Foundations," by Kimball R. Garland, *Engineering News-Record*, September 26, 1935, p. 427.

TRENT R. DAMES,¹¹ JUN. AM. SOC. C. E. (by letter).^{11a}—The most significant single statement in Professor Tschebotareff's paper is his conclusion that "Results obtained from observations in one locality are not necessarily valid in other localities" (see heading, "Conclusions"). This truth deserves more emphasis than it has yet received and is a fact of which any one engaged in developing general theories in the new field of Soil Mechanics should be always mindful.

Most of the new theories are the result of studies made largely on the glacial clays of Northeastern United States and Europe. Several years of effort on the part of the writer and an associate, W. W. Moore, Jun. Am. Soc. C. E., to apply many of them without modification to problems involving the foundation and embankment soils of the semi-arid regions of Southwestern United States have, in general, met with failure. Different factors assume importance; different soil tests become desirable; and different methods of analysis seem necessary.

Believing the soils of the Nile Valley to be somewhat akin in character and behavior to certain of those with which he has dealt, the writer is encouraged to enter this discussion and contribute what he has been able to learn of the behavior of soils of reasonably similar geologic background. This discussion will take the form of certain comments on the testing methods and the advancing of alternate interpretations of such settlement data as the author has found it possible to include.

Building I, Fig. 1.—In common with all of the other structures cited, Building I raises some questions as to testing procedure. The liquid and plastic limit tests admittedly "illustrate the potential properties of the basic soil material in a disturbed state" (see heading, "Results of the Settlement Studies"), and accordingly have value for such purposes as routine tests for highway subgrades. Their value is questionable, however, when they are applied to foundation soils which, in most cases, are used only in the undisturbed state. The use of these tests to subdivide "silt" and "clay" is novel to the United States where a division based on mechanical analysis is popular. Both methods are cumbersome and difficult of ready application; and a flexible classification based on appearance, texture, and local behavior has been found equally, if not more, effective. For foundation purposes, the effort spent on these and similar tests might better be applied to more thorough field explorations and on tests which describe the behavior of undisturbed soil under load, as, for example, shear and consolidation tests on core samples extracted from the borings.

Building II, Fig. 2.—In discussing Building II, Professor Tschebotareff, in common with nearly all modern writers, condemns the load test. In this case the accuracy of the readings taken is not sufficient to "close the case" and justify this condemnation. Admittedly the load test, as generally performed, clouds the picture and frequently breeds false security, but when properly conducted, it is a valuable aid to the foundation engineer. In the case in question, if the test area and water tower can be assumed to settle roughly in proportion to their diameter (no data on which to base an opinion being available), the

¹¹ Dames and Moore, Los Angeles, Calif.

^{11a} Received by the Secretary December 12, 1938.

settlement of the test area during the probable time of test (not stated) very possibly would not have been detected.

To get results from a load test, the soil and location at which it is made must be chosen carefully, not necessarily at foundation depth. The increments of load should be known to within 5%; the settlements should be measured (for test areas of, say, 2 sq ft to 10 sq ft) to 0.001 in.; and sufficient time should be allowed for the test footing under each increment of loading to achieve equilibrium. The shape of the curve is frequently of more importance than the magnitude of the settlements, and it indicates to the engineer who is familiar with its interpretation many characteristics that no laboratory test will describe.

The author describes the errors in computed settlements resulting from an analysis dependent on compression tests made on swelling clays and silts that are sampled with difficulty. The load test can be, and has been, used to obtain an experimental ratio by which the settlement computations may be multiplied to predict the settlements of a structure more accurately. (The adjustment of settlement computations by means of a ratio obtained in this manner frequently formed part of the settlement computations reported by Mr. Moore and the writer.)¹²

The record of Building II is used to justify the statement: "No Decrease of Settlement Was Observed as a Result of Excavation" (see heading, "Results of Settlement Studies"). It is true that Building II has settled about 1 in. during the three years of record, but no data are introduced to disprove the logical premise that this structure would have settled more than 1 in. if founded at, or near, the surface of the ground. Comparison with the only other building founded on soil 2 000 yr old (Building I) is futile because, in this case also, moderate excavation was resorted to and more particularly the soil had undoubtedly been pre-stressed by the considerable load of the old water tank, the bottom of which was utilized for the foundation of the new structure.

Of the buildings on soil 800 yr old, Building III is on spread footings and is not comparable as to load over an extended area. The spread footings of Building IV so nearly cover the area as to approach a mat in behavior. A similar sequence of soils both as to classification and description by the physical tests given exists under both of Sites II and IV. In each case the sand is at 40 ft; and the unit loads on the foundations are the same. The settlements of Building IV, founded near the ground surface, are more than three times those of Building II, founded at a depth of 18 ft, and occur during a somewhat shorter period of record.

Another mat of similar size, that of Building IX, founded near the surface and underlain by the same quantity of compressible soil as Building II, settled six times as much under one-half the unit load, or a factor of 12 to 1. The uppermost silt underlying Building IX is stated to be about 100 yr old, and the lower soils are of more recent age than those under Building II. However, in the absence of tests of the compressibility of these recent soils, it would seem unwarranted to assume an average compressibility coefficient in one case of twelve times the other, particularly when the proof of the case against excavation as a means of reducing settlements rests on such an assumption.

¹² *Civil Engineering*, January, 1939, p. 41.

Building III, Figs. 3 and 4.—The settlement of Building III at an undiminished rate from the middle of 1935 through 1937 would seem to warrant an attempt at explanation. On many California soils such a settlement curve would probably indicate that incipient shearing failure has occurred, and that a moderate degree of plastic flow of the over-loaded clay from beneath the spread foundations of this structure is occurring.

Building IV, Figs. 5 and 6.—This building was discussed under Building II as an example of lack of excavation. As in Building III, the continued, undiminished rate of settlement since the middle of 1936 could be interpreted as caused, at least in part, by shearing failure and the resulting plastic flow of over-loaded soil from beneath the foundations of this building.

Building T, Figs. 13 and 14.—For convenience of discussion, Building T will be considered before any of the other buildings on pile foundations. The writer is unfamiliar with the type of piling used in these Egyptian structures as it is seldom, if ever, used on the Pacific Coast of the United States. For this reason he will have to assume that the diameters of these piles (which are not given by the author) are nearly comparable to the diameters of the driven or cast-in-place piles of his experience.

The individual piles of all of the buildings described in this paper seem more heavily loaded than the soil conditions warrant. Tests of the shearing strength of the soil which would permit a determination of the load at which individual piles would begin to penetrate the soil about them are apparently not available. The load tests presented in Fig. 14(c), however, most certainly indicate that this penetration is occurring at loads in excess of 40 tons, possibly even at lower loads, the settlement scale being too small to describe the shape of the curve below this point. On several occasions the writer has found that the point at which the curve of a pile load test departs from an approximate straight line agrees remarkably well with a determination based on a summation of the circumferential area of the pile times the shearing strength of the soil penetrated and the bearing on the pile point. The spacing of the piles when grouped in clusters is not stated, but if it is at all similar to the practice in California they are so close that the circumference of the group times the shearing strength of the soil will give a lower value than 40 tons per pile for the load at which penetration of the group occurs.

Additional support for the belief that the settlement of Building T has been largely the result of individual pile failure and not due to the compression of the 10 ft of dark clay between the pile points and the sand is furnished by the following reasoning: This dark clay is stated to be more than 800 yr old and to have been subjected to compaction and consolidation by the repeated drying and flooding by the River Nile through the intervening centuries. Accordingly, it is reasonable to assume, from the tabulation following Equation (1), that the compression coefficient for stiff brown clay (0.007) is reasonably representative of this material and not the value given for plastic dark clay (0.070), which, in the absence of test data on either Fig. 13 or 14, was presumably obtained by working backward from the settlements of the structure. Using the value of the compression coefficient (0.007) the maximum load of 17 tons per sq m ($= 1.6$ tons per sq ft), and the thickness of the dark clay (10 ft), the compression

in the clay layer becomes $1\frac{3}{8}$ in. leaving the remainder of the 10 in. of settlement recorded to date to be attributed to individual or group pile penetration.

Building V, Fig. 7.—The piles in Building V are more moderately loaded than those in Building T, but in view of the 800-yr age of the soils beneath the piling and the magnitude of the settlement (2.5 in.) to the middle of 1937, about one-half the present settlement may need to be explained by penetration of the piling. If this is true, this building may continue to settle appreciably for some years to come.

Building VI, Fig. 8.—The average load of 64 tons per pile seems excessive, even when the 6-ft to 8-ft penetration of the 27-ft piles into the very fine sand is considered. The shape of the time-settlement curve and its failure to quickly become horizontally asymptotic would indicate that shearing failure and flow of the soil is occurring. The attribution of any appreciable part of this settlement to the effect of gradual consolidation of the overlying clay and silt layers and the resulting decrease of the frictional support they temporarily provided does not seem reasonable. In view of its shearing rigidity being necessarily considerably less than that of the sand, the brown silt layer could scarcely have been carrying any large proportion of the total load at any time; and, in the three years of record, this 12-ft or 13-ft layer could scarcely have consolidated under the weight of the superimposed silt and "old" fill as much as the 1.25 in. settlement recorded.

Lacking data as to the occupancy and location of this building the writer would hazard a guess that the continuation of the settlements may be aided by elevator or traffic vibrations transmitted by the piling to the over-stressed sand. Similar examples have come to his notice in the case of buildings founded on sand which has been over-loaded by the foundations. A building occupied by a publishing establishment showed visible distress only in the vicinity of a heavy press. In some buildings cracks have been observed only in the bays immediately adjacent to the elevator shafts.

Building VII, Fig. 9.—Inasmuch as the piles seem primarily end-bearing piles and apparently would not receive any large proportion of their support from the less than 100-yr old silt and clay deposits above the sand, the pronounced irregularities of settlement cannot be primarily the result of irregularities in the compressibility of the soil. Vibrations are again hazarded as a possible explanation. Included in this category must be considered any vibrations that occur during the placing of the piling and during construction. For example, the state of stress in the compacted and densified sand under the first piles to be placed might have been released by vibrations occasioned during the placing of subsequent piles. The remaining three buildings are similar to those previously discussed.

Although the writer has taken a different point of view than the author and has taken every opportunity to present and advance a different interpretation of the data, he appreciates, thoroughly, having had the opportunity to study such a comprehensive paper and believes that Professor Tschebotareff deserves the highest praise for having so completely made the results of his observations available to the profession.

EDWIN J. BEUGLER,¹³ M. AM. SOC. C. E. (by letter).^{13a}—A distinct service has been performed by Professor Tschebotareff in presenting unusually complete data as to settlements occurring with various types of foundations on sedimentary formations under an imposed unit loading. The data, largely in graphic form, afford a clear picture of cause and effect. Several types of footings are shown with a diversity of soils and intermittent ground-water levels.

Of prime importance is a distinction between the relative settlement during construction and that occurring later under the completed structure. The former is due to compression, shrinkage, and more or less elastic reaction, and ceases when the building is finished, or shortly thereafter, provided the unit loading of the soil has been selected conservatively. Conversely, continued progressive settlement of the completed structure indicates over-loading and ultimate failure. If the rate of progressive settlement is decreasing it is possible, in most cases, to predict the time of ultimate cessation.

The time-settlement curves given in Fig. 9 (Building VII) shows a well-designed foundation for the situation involved, and illustrates the foregoing statements. Loading as construction proceeded caused a maximum compression of about 1.3 in. After the structure was completed no appreciable settlement occurred. The action may be likened to placing a load on a spring scale which is compressed proportionately to the applied load and thereafter remains stationary under a constant load.

Much of the confusing information concerning settlements has been due to overlooking this important phase. For example, the progress report of the Special Committee on Earths and Foundations, presented in 1933, pictures a settlement record of the Washington Monument¹⁴ from 1879 to that date, amounting to 5.5 in. and "probably still slowly settling." Analysis of the record shows that 4.5 in. of this settlement occurred between 1879 and 1885 during construction, which added 400 ft to the height of the shaft as completed in 1885. The remaining settlement of only 1 in. under a constant load during nearly 50 yr, it must be granted, indicates a fairly stable structure, and a monument to the engineers who so skillfully underpinned the old foundation, as well as a monument to the hero of 1776.

The self-explanatory diagrams may enable one to determine offhand the cause for failure or inadequacy. From the time-settlement curves for the various buildings it may be concluded that over-loading is responsible for most of the progressive settlement. Some was due to the fact that piles did not penetrate to firm material, or that they failed to distribute the load in other respects.

The author states that no decrease of settlement was observed as a result of excavation (heading "Results of the Settlement Studies"). This was doubtless true for the case cited. On the other hand, one instance in the writer's experience is worthy of note. In 1919 an important structure was settling progressively at one end and remaining stationary at the other. It was supported on piles driven to refusal. Rock was 40 ft below grade at the undisturbed section

¹³ Cons. Engr., Cheshire, Conn.

^{13a} Received by the Secretary December 31, 1938.

¹⁴ *Proceedings*, Am. Soc. C. E., May, 1933, Fig. 34, p. 813.

and 170 ft below with intervening sedimentary strata, including a band of soft clay, at the depressed end. There were no cracks in the structure. The settlement of 18 in. was stopped by excavating earth within and around the building, the outside area being replaced by light reinforced cellular construction. The load on the area was reduced about one-third before the settlement ceased. The structure has since remained stationary.

The author states that no useful relation could be established between load-test results and the settlement of full-sized structures. The meager data given regarding the method of making such tests precludes comment as to other methods and deductions therefrom. Reference to this and other matters has been made by the writer elsewhere.¹⁵

¹⁵ *Proceedings*, Am. Soc. C. E., October, 1933, p. 1358; also, *The Military Engineer*, July-August, 1933, pp. 326-327.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

BEAM CONSTANTS FOR CONTINUOUS TRUSSES AND BEAMS

Discussion

BY C. W. DEANS, ESQ.

C. W. DEANS,⁸ ESQ. (by letter).^{8a}—The writer favors the use of elastic weights for the determination of deflections of trusses, arches, and beams of variable section and finds Mr. Epps' paper most instructive and essentially useful for design office calculations.

While checking over the theory and detailed calculations of Mr. Epps' presentation it was felt that an actual demonstration of the proof of the theory by Maxwell's General Law of Reciprocal Energy Components might be helpful to those who desire to fix these ideas firmly in mind. In this law, if two states of stress for a structure are considered, the sum of the products of the forces for the first state (P_1) and the displacements, at corresponding points for the second state (y_2), is equal to the sum of the products of the forces of the second state (P_2) and the corresponding displacements for the first state (y_1). Written in equation form this is:

$$\Sigma (P_1 y_2) = \Sigma (P_2 y_1) \dots \dots \dots (3)$$

Beam $A B$, Fig. 6, is influenced by two states of stress and Equation (3) is written for the active forces only:

$$(-F_{AB})(\theta_A = +1) + (P = 1)y = M_{AB}(\theta_A = 0) \dots \dots \dots (4)$$

Therefore, $y = F_{AB}$, which demonstrates that the deflections of a beam with unit rotation at one end are the influence ordinates for the fixed-end moment at the rotated end. In Fig. 7, Beam $A B$ is again shown under two states of stress. From Equation (3):

$$M_{AB}(\theta_A = 0) = m_{AB}(\theta_A = +1) + m_{BA}(-\theta_B) \dots \dots \dots (5)$$

Therefore, $\theta_B = \frac{m_{AB}}{m_{BA}} = r_{BA}$, which demonstrates that the rotation at the far

NOTE.—This paper by George L. Epps, Jun. Am. Soc. C. E., was published in October, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

⁸ Estimator, Hamilton Bridge, Western, Ltd., Vancouver, B. C., Canada.

^{8a} Received by the Secretary, November 19, 1938.

end of a beam is the carry-over factor (r_{BA}) for moments from the far end to the near end of the beam.

A practice problem illustrating the applications of Mr. Epps' method to a beam of variable section is afforded by a beam analyzed⁹ previously by G. E. Large, Assoc. M. Am. Soc. C. E. The preliminary calculations are already made by Professor Large and the remainder necessary to arrive at the influence

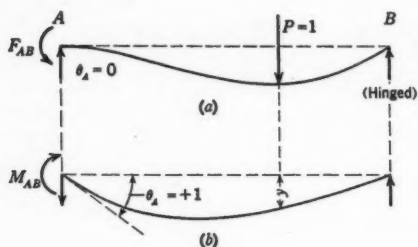


FIG. 6

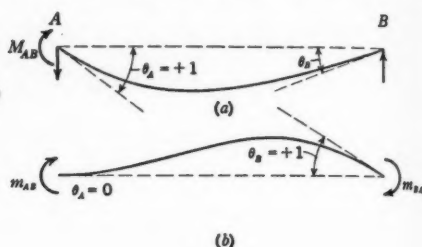


FIG. 7

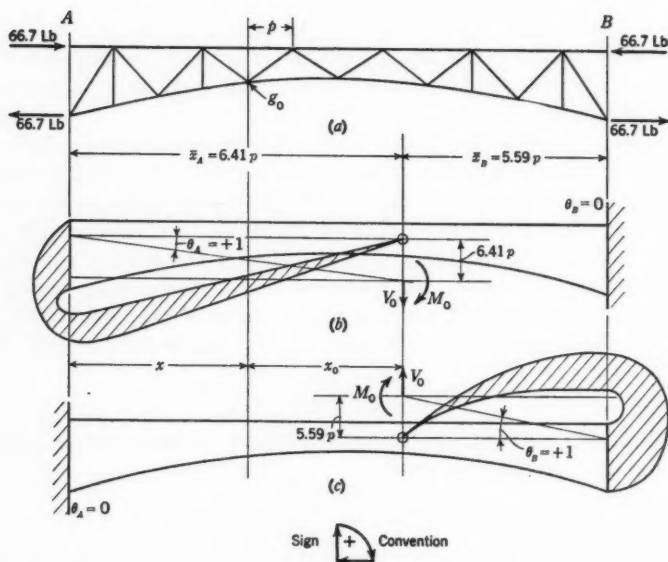


FIG. 8

ordinates for fixed-end moments are quickly obtainable. This essential practice will help one to follow more readily the computations required for the truss analyzed by Mr. Epps.

The writer has favored the use of the column analogy,¹⁰ or the elastic center method for the determination of beam constants; and thus he set to work to apply a moment, $M_0 = 1\,000$ ft-lb, to each end of Fig. 3 as shown in Fig. 8(a);

⁹ Transactions, Am. Soc. C. E., Vol. 96 (1932), p. 105.

¹⁰ "The Column Analogy," by Hardy Cross, M. Am. Soc. C. E., Bulletin No. 115, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

thus: $\Delta_{M_0=1} = \Sigma g_0$; $\Delta_{V_0=1} = \Sigma (g_0 x_0^2)$; $M_0 = \frac{\Delta_{M_0}}{\Delta_{M_0=1}}$; and, $V_0 = \frac{\Delta_{V_0}}{\Delta_{V_0=1}}$. Referring to Fig. 8(b): $\Delta_{M_0} = 1$; $\Delta_{V_0} = -6.41$ p; $M_{AB} = M_0 + \bar{x}_A V_0$; $M_{BA} = M_0 + \bar{x}_B V_0$; and $r_{AB} = \frac{M_{BA}}{M_{AB}}$. Finally, referring to Fig. 8(c): $\Delta_{M_0} = +1$; $\Delta_{V_0} = +5.59$ p; $M_{BA} = M_0 + \bar{x}_B V_0$; $M_{AB} = M_0 + \bar{x}_A V_0$; and, $r_{BA} = \frac{M_{AB}}{M_{BA}}$.

Arrangements similar to Table 1 were made for $S_0 = \frac{M_0}{l}$ (in which l = the lever arm of the member), L , A , $\frac{L}{A}$, r , $\frac{L}{Ar}$, and the partial elastic weights, $\frac{S_0 L}{Ar} = g_0'$. The elastic weights (g_0) were determined by summing g_0' for each panel point; and, thus it was found that the elastic center was 6.41 panel lengths from the left support at Panel Point 8. The next step is to compute Σg_0 and $\Sigma (g_0 x_0^2)$, which is the moment of inertia of the analogous column for the column analogy, or the vertical displacement of O , of the rigid bracket, AO (with a vertical load $V_0 = +1$ at Point 0) for the elastic center method. These values in conjunction with V_0 and M_0 in Figs. 8(b) and 8(c) give stiffness factors and carry-over factors for both ends of the truss. The value of the carry-over factor from the far end (Point 20, Fig. 3) to the near end (Point 8, Fig. 3), thus determined, was 0.51, which is almost identical with that derived by Mr. Epps.

The influence lines for fixed-end moments are found from the values of V_0 and M_0 in Figs. 8(b) and 8(c) by calculating the cumulative displacements in the same manner as Mr. Epps in Table 2, Columns (6), (7), and (8). These values of stiffness factors, carry-over factors, and fixed-end moments can be used directly in the moment distribution and the slope-deflection methods.

The foregoing illustrates the fact that the moment transfer from span to span depends primarily on the proportions of the chord members and the depth variation of the trusses. The web members come into play mostly on account of the vertical shears caused by the loads and show up in the fixed-end moment calculations. It will save time in the calculation of Table 2 for both ends of the span to tabulate the values of $\frac{L}{Ar}$ to be used with each of the two stress conditions.

With the elastic center method the fixed-end moments for beams having the far ends hinged are determined from the following equations:

$$H_B = C_B - r_{AB} C_A \dots \dots \dots (6a)$$

and,

$$H_A = C_A - r_{BA} C_B \dots \dots \dots (6b)$$

and the stiffness must be multiplied by $1 - r_{AB} r_{BA}$.

The author's method is very straightforward and even though the values found must be corrected on account of the far ends being considered hinged, only two sets of calculations (as in Tables 1 and 2) must be made—one for each end. In the suggestions offered by the writer three sets must be made,

one for determining the position of the elastic center and two for the influence lines for fixed-end moments at each end. A full explanation has been made in this discussion because it was felt that a complete study would help in obtaining a clearer picture of the entire problem.

Mr. Epps is to be complimented on his choice of method, which is most efficient in that it saves design office time, and on his masterly, all-inclusive presentation. It would be interesting for Mr. Epps to comment on the feasibility of using multiple-tied, trussed arches on roller supports or multiple-trussed arches on flexible, trussed towers, and on the application of his method to their analysis.

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DISCUSSIONS

SOLUTION OF EQUATIONS IN STRUCTURAL ANALYSIS BY CONVERGING INCREMENTS

Discussion

BY L. E. GRINTER, M. AM. SOC. C. E.

L. E. GRINTER,¹⁰ M. AM. SOC. C. E. (by letter).^{10a}—This paper seems to bring into usable form a general method of solving simultaneous equations in structural analysis. Professor Dell sees very clearly the advantage of standardizing such solutions with the objective of eliminating errors and of shortening the labor involved.

The most interesting part of the paper, in the writer's opinion, is centered around the solution given in Fig. 4. The author likens this solution to the method of balancing moments, but actually the correspondence seems to be no deeper than a mere similarity of appearance. There is an interesting comparison of the basic conceptions involved that seems important enough to mention.

Within recent years there have been two quite different systems of analysis of indeterminate structures: (1) The classical methods dependent upon the mathematical solution of simultaneous equations; and (2) the modern methods dependent upon a physical picture of the action of the structure that can be expressed as a series convergence of moments and shears, or of angles and deflections. The two systems seem distinct and separate. Certainly the development of the modern methods, started by Hardy Cross, M. Am. Soc. C. E., occurred as a sharp cleavage from the mathematical procedure of the classical methods. Neither moment distribution nor the method of balancing angle changes was developed as a mathematical tool for solving a group of simultaneous equations. Each was developed as a physical procedure that gave rise to a solution of the problem without reference to simultaneous equations.

The calculations of Fig. 4 seem to fit in between the classical and the modern

NOTE.—The paper by George H. Dell, Assoc. M. Am. Soc. C. E., was published in October, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1939, by A. Floris, Esq.

¹⁰ Dean, Graduate Div., and Director of Civ. Eng., Armour Inst. of Technology, Chicago, Ill.

^{10a} Received by the Secretary December 12, 1938.

methods of analysis. It is strange that their presentation should follow rather than precede the presentation of the modern methods themselves. This merely illustrates what a distinct advance the procedure of balancing moments represented. In one tremendous sweep the classical methods were pushed aside and the modern methods were introduced. There was no intermediate step such as Fig. 4 might have represented.

Fig. 4, however, does not reveal a very clear physical picture for the computations recorded thereon. Equation (1) expresses as its right-hand member the important quantities used in Fig. 4. That is, the expression $\frac{6 \bar{x}_1 A_1 K_2}{L_1^2} + \frac{6 \bar{x}_2 A_2 K_1}{L_2^2}$ gives rise to the quantities recorded at the top of each column of numerals in Fig. 4; but this expression is actually equal to $6 (\theta_1 + \theta_2) K_1 K_2$, in which θ_1 and θ_2 are adjacent end slopes of the simple spans meeting at the joint considered. Thus, the quantity to be used in balancing is neither a moment nor an angle; it is the sum of two angles multiplied by the product of two stiffness factors. It does not seem hopeful to attempt to attach a very useful physical significance to such a quantity as this.

The argument as to whether the desirable solution of a given problem is mathematical or physical can never be settled. A purely mathematical solution such as the author suggests is useful to persons who visualize with difficulty. A physical procedure, which is merely doing with arithmetic what could be done to a model of the structure in the laboratory, is advantageous primarily to the person with imagination.

The writer sees the physical concept in analysis as being most useful when there is need for extension of the method to new cases, new arrangements or new

2 EK	33	28.8	44.8	42	
$\frac{\Delta}{A}$ (10.5)	$\frac{\Delta}{B}$ (6)	$\frac{\Delta}{C}$ (8.4)	$\frac{\Delta}{D}$ (14)	$\frac{\Delta}{E}$ (7)	$\frac{\Delta}{F}$
	-238	-204	-141	-157	
	+ 52	+ 61	+ 70	+ 11	
		+ 10	+ 29	- 4	
		- 13	+ 3		
		- 4	+ 1	- 1	
	+ 3	- 1			
Sums	-183	-151	-39	-151	
Divided by 2 EK =					
Moments =	- 55	- 53	- 9	- 36	

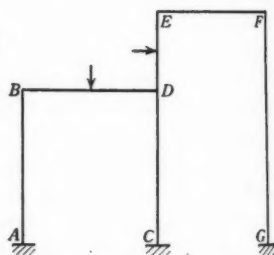
FIG. 6.—ABBREVIATED COMPUTATIONS OF $(\theta_1 + \theta_2) K_1 K_2$ 

FIG. 7.—COMPLEX BENT

structures. One can always "fall back" upon the fact that the arithmetical process merely states, step by step, the physical treatment that could be accorded a model of the structure in the laboratory. There is a one-to-one correspondence between the recorded arithmetical steps and the physical adjustments visualized for the model.

If the author is interested in saving time in analysis there is no reason why his calculations of Fig. 4 should not be reduced to those given in Fig. 6. In this form there will actually be fewer recorded intermediate results than would be used in balancing moments or angles, but an extension to a more complex case

will not be self-evident. For example, if the problem is to analyze the bent of Fig. 7, what should be the initial factors at the joint *D*? Should the carry-over factors be the same as in Fig. 6, and how is the shear balance to be effected? These questions must be answered from a study of the simultaneous equations involved, and the answer is not self-evident.

The calculations of Fig. 6 have been simplified by a reduction of the number of significant figures and by a change of signs. Three significant figures obtainable from an ordinary slide-rule are, in the writer's experience, almost always adequate for design purposes. The results of Fig. 6 are expected to be in error by unity in the last figure, but the influence upon important moments is not significant. It is unlikely that the small moment of -9 at Point *D* would have any important influence upon the design. The change of sign for the carry-over values was adopted so that the column addition could be made for all recorded values and so that numbers need not be repeated. The result is a convenient and rapid automatic procedure for analyzing the special case of a continuous beam with simply supported ends. Whether it can be developed to be equally useful for other types of beams and for continuous frames is another matter. The author has performed a service by describing what might well have been an intermediate step between the classical and the modern methods of analysis although his actual procedure remains primarily a tool for use with the classical methods.

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DISCUSSIONS

TRANSPORTATION DEVELOPMENTS IN THE UNITED STATES

Discussion

BY MESSRS. MILTON HARRIS, AND RALPH BUDD

MILTON HARRIS,²² ASSOC. M. AM. SOC. C. E. (by letter).^{22a}—The rise and fall of the American railroad has been treated very thoroughly by Mr. Lavis. After justifying the railroad as the major national transportation facility for long-haul freight and showing the relationship of highway transport to that phase of the problem he lightly touches on aviation, waterways, and pipe lines, as also belonging in the transportation picture.

Improvement in transportation facilities, he notes, has been largely financed by Government aid, except that railroads have not been a beneficiary (of late), but have been beset by governmental regulations as to taxes, rates, and wages. The solution, according to Mr. Lavis, is Federal aid to put the railroads on a par with Government-financed highways, waterways, and airways.

In drawing the deplorable picture of the condition of the railroads of the United States, Mr. Lavis has omitted one vital figure that stalks the background from the beginning of railroad history. To-day, this figure stands out as the foremost agent responsible for "sick" railroads. That figure is "management." This word may be subdivided into financial and operations management; the first is under indictment, and the latter is responsible for the finest system of railroads in the world.

Management that bred the expression "the public be damned" soon led that self-same public to devise regulations to protect their own interests from the depredations of voracious financiers who were not aware that they were in the transportation business. Therein lies the main difference between highway and rail transportation. The highways are not listed on the stock exchange. True, they are in many cases under the control of political pressure groups, but those groups, by their own greedy tactics, are soon ousted, and it is a fair sign

NOTE.—This paper by Fred Lavis, M. Am. Soc. C. E., was presented at the Annual Convention, Salt Lake City, Utah, on July 20, 1938, as part of the Symposium on Transportation, and was published in November, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

²² Associate Highway Engr., State Div. of Highways, Sacramento, Calif.

^{22a} Received by the Secretary December 13, 1938.

of progress that highway engineers are now becoming aware of their heritage in that they are becoming "traffic-minded"—that highways are not merely pavements, structures, and grading quantities, but are vital elements of the transportation problem and should be so designed and operated with that in mind.

About 20 or 25 yr ago railroad managements were presented with many, many miles of feeder road-bed. It was laid in their laps practically free of cost and all they were asked to do was to provide the rolling stock. Because these short-sighted gentlemen could see no ballast, ties, or rail winding into the hinterland, they lay back and allowed a younger, keener breed of executives to populate the highways with trucks and buses that stole the short-haul business. Too late did railroad management "let out a howl to high heaven" that they were robbed.

Lack of money is not entirely the pressing problem of the moment for the railroads. At one time in their history, it was the very least of their troubles; but unredeemed bond issues, structures, plant, and equipment that have paid themselves out many times over, but still appear on the books as capital investment (combined with resulting inflexible rate structures forced on the companies as the result of their own shortsighted policies) have now tied the railroads into a "balled up mess of red tape." Money will not cut the way out except in the hands of modern executives who realize the place of the railroad in the modern transportation scheme and can so co-ordinate that element to the whole that the entire structure is basically in tune with modern needs and economics.

"Railroaders" to-day are antiquated. They "railroad" like their grandfathers did. New blood is needed—new thoughts and the application of the newer sciences. How many college graduates do the railroads absorb each year? What possible incentive under the present regime is held out for such men that other major industries seek so avidly? Where are the railroad research laboratories? What contributions have the railroads recently made to civilization where the crying need of the hour is more economical distribution of the products of labor?

These are the questions to be answered by railroad management. With a transfusion of modern executive blood into its system, and a lightening of governmental regulation, the railroad problem will fit itself quietly into the complete transportation picture and its financial worries will not affect civic life to the extent that they do at present. That the railroad has a definite place in the distributive system, no one can deny, but until railroad executives realize that they are in the transportation business instead of in the railroad business, Federal monetary aid to help them in their dilemma will merely be "pouring sand down a rat hole." Reference to Table 3 will clarify this viewpoint. The sum of Columns (3), (4), and (5) (Table 3) equals the total railroad capital outstanding. Careful year-by-year study of the relationship of the elements of capitalization against book value should indicate the mortgage that railroad management has allowed to accrue against their plant without change in their operating methods. The heritage of past financial evils rides the railroads of to-day.

TABLE 3.—STATEMENT NO. 53 OF THE FIFTEENTH ANNUAL REPORT OF
THE STATISTICS OF RAILWAYS IN THE UNITED STATES FOR
1936 BUREAU OF STATISTICS, IN DOLLARS

Year ended December 31	Investment in road and equip- ment—book value (Classes I, II, III, lessor, and proprietary companies)	Common stock outstanding	Preferred stock outstanding	Funded debt outstanding	Net capitalization equals total capital less sum of railroad stocks and bonds owned by railroads
(1)	(2)	(3)	(4)	(5)	(6)
1923	21 372 858 000	7 398 000 000	1 852 000 000	13 589 000 000	17 810 000 000
1924	22 182 287 000	7 539 000 000	1 935 000 000	14 162 000 000	18 202 000 000
1925	23 217 209 000	7 602 000 000	1 937 000 000	14 105 000 000	18 191 000 000
1926	23 880 740 000	7 560 000 000	1 925 000 000	14 192 000 000	18 234 000 000
1927	24 453 871 000	7 683 000 000	1 980 000 000	13 951 000 000	18 137 000 000
1928	24 875 984 000	7 809 000 000	2 034 000 000	13 904 000 000	18 511 000 000
1929	25 465 036 000	7 853 000 000	2 065 000 000	14 065 000 000	18 680 000 000
1930	26 051 000 000	8 009 000 000	2 074 000 000	14 248 000 000	19 066 000 000
1931	26 094 899 000	8 031 000 000	2 049 000 000	14 264 000 000	18 941 000 000
1932	26 086 991 000	8 067 000 000	2 047 000 000	14 723 000 000	18 894 000 000
1933	25 901 962 000	8 057 000 000	2 042 000 000	14 624 000 000	18 831 000 000
1934	25 681 608 000	7 994 000 000	2 044 000 000	14 532 000 000	18 653 000 000
1935	25 500 465 000	7 987 000 000	2 036 000 000	14 224 000 000	18 342 000 000
1936	25 432 388 000	7 993 000 000	2 036 000 000	13 974 000 000	18 336 000 000

The railroads pioneered the opening of vast domains of natural resources of virgin country which were soon settled and producing traffic which increased with the years. Then came stabilization of population and industry, followed by the seeking of transportation at a lower price engendered by competition. Highway transport filled this need to a certain extent and decentralization of industry followed to decrease further the rail revenues. As a result there may be a surplus of rail facilities, as Mr. Lavis states, but is that a reason why the consumer or taxpayer should be forced to keep these dying tentacles alive? Would it not be more economical to abandon this surplus, write off the investment and replace it with a cheaper form of transportation plant? Possibly this would lead to a profitable organization, selling transportation at the lowest possible value by using the railroads as a back-bone and the more profitable self-sustaining highways as feeders, aided by pipe lines, airways, and water transport, all co-ordinated and consolidated into a national transportation system.

RALPH BUDD,²³ M. AM. SOC. C. E. (by letter).^{23a}—The pity is that the information in Mr. Lavis' paper cannot be more widely disseminated and brought home to the general public. Lack of general understanding of the facts, so ably marshalled by him, is one of the primary factors in the so-called railroad problem.

Many thoughtful persons believe that substantial portions of the investments in railways already have been confiscated through low rates and high wages. Almost one-third of the railroad mileage is being operated under trusteeship, nearly one-half did not earn operating expenses and taxes in 1938, about nine-tenths failed to earn taxes, operating expenses, and fixed charges,

²³ Pres., C. B. & Q. R. R., Chicago, Ill.

^{23a} Received by the Secretary December 15, 1938.

and only about one-tenth earned anything for improvements or dividends. Education of the public must come from the disinterested studies of transportation experts and not from the clamor of minority pressure groups which have selfish interests.

Mr. Lavis' suggestion that there should be intensive and centralized research will meet with general approval, but more research is now (1939) being conducted than is generally recognized. The Association of American Railroads has done some work, although the extent of its research has been disappointing. Manufacturers of steam and Diesel locomotives, manufacturers of cars, machinery, and supplies, and the individual railroads have contributed to far-reaching improvements made in recent years.

With great deference to Mr. Lavis, the writer is not quite content with his statement that Diesel engines have not yet progressed much beyond the experimental stage for general railroad traction. The Diesels have demonstrated their efficiency and economy in switching service and in passenger train service, especially on long-distance high-speed schedules. Coincidentally, the newer steam locomotives are greatly improved in design and capacity. Recently some very remarkable results have been obtained by the use of motion pictures in the study of "pound" on track by locomotive driving wheels at high speeds—either actual or slippage speeds. Space forbids enumeration of the many improvements that have been made possible in the last few years by intensive study and research. The daily operation of about 240 000 miles of railroad provides a laboratory in which a vast total of improved methods and devices are being developed all the time. Much of this development is not spectacular or widely publicized, but it is going on day in and day out as the result of the thought and ingenuity which hundreds of railway engineers are devoting to their work.

As Mr. Lavis states, there is no lack of brains, knowledge, or engineering and scientific ability; what is needed is money. It would be a splendid thing if the railroads could afford to establish a nation-wide railway laboratory, in charge of a staff of skilled engineers. However, there are several financial aspects of that problem. It is not alone the initial cost of establishing such a research laboratory, or the expense of maintaining it, which makes such an investment difficult under the present financial condition of the railroads; the utilization of improvements frequently is disappointing from the standpoint of direct financial return on the investment. For one thing, there are many archaic rules respecting pay and working conditions which are so little adaptable to improved devices that no actual saving can be accomplished thereby. The development of new devices and of light-weight metal alloys of great strength has opened new vistas of building, and operating, light-weight locomotives and cars, which would reduce the ratio of dead weight to total train weight, reduce wear and tear on roadway and rolling stock, and permit faster and safer operation. However, existing equipment which is in serviceable condition cannot be scrapped immediately and replacements can be made only gradually. In the public interest it is desirable to make such replacements largely through depreciation accounts rather than by increasing capital accounts. To do this within a reasonable time requires a higher standard of earnings than has

been permitted to the railroads in the past. Frequently there are opportunities for the railroads to make expenditures to take advantage of new ideas, with prospect of large return upon the investment; but as investments in railroads already are so large, and the return so meager, conservative management cannot consistently take any unnecessary risk with the capital available. If private enterprise is to provide necessary funds with which to take advantage of improvements and developments in railroading, there must be greater prospect than in the past that those investing the money will have some chance to share in the benefit of the improvements. It is not enough that the shippers and employees shall be benefited.

Some part of earnings must be paid for the use of money which makes earnings possible. Equipment trust certificates, at relatively low rates of interest, are well regarded by investors because they carry reasonable assurance of security of principal and interest.

Mr. Lavis suggests that the Government should provide funds for the rehabilitation of the railroads, either by loans (see heading, "The Solution: Government to Furnish Capital and Credit to Railways") or by gifts (heading, "The Money Question: Government Funds and Credit"). For a number of years the railroads have been borrowing money from the Government, chiefly through the Reconstruction Finance Corporation, and such loans, designed to tide over railroads that could not obtain money elsewhere, have been of great assistance; but such loans are merely temporary assistance—not a cure. In so far as the aggregate of railroad debt is increased in that manner, the day of reckoning is simply postponed; and, ordinarily, increasing the debt load reduces the credit.

Mr. Lavis makes a plausible statement of reasons why the Government well might grant the railroads a subsidy of several billions of dollars, in view of the much greater financial aid it is giving to other agencies of transportation. The writer does not think that Government subsidy should be considered as anything but a last resort. The capitalization of the railroads as a whole is below the valuations made by the Interstate Commerce Commission and, on the whole, the fixed indebtedness is not excessive. The railroad problem would not be solved by reducing fixed charges through writing off losses of investors, nor by Government loans without interest, nor by subsidies, unless it is to be solved through resort to Government ownership. If investors are to put new money into the railroad industry, what is needed is an increase in net railway operating income and elimination of unnecessary expenditures, not devices for expediting the so-called "wringing process."

To increase net railway operating income, the rate-making rule of the Transportation Act of 1920, which was practically eliminated by the Act of 1933, should be restored. In substance, the 1920 rate-making rule directed the Interstate Commerce Commission, in regulating rates, to permit the carriers as a whole, or in groups selected by the Commission, to earn an aggregate annual net railway operating income equal to a fair return on the aggregate value of the railway property used in transportation. In 1933, the statute was changed so as to omit this definite requirement and to instruct the Commission to give due consideration, among other factors, to the effect of rates on the

movement of traffic. This new provision has been interpreted by the Commission as a direction to consider many matters aside from transportation conditions in determining the reasonableness of rates, and to substitute its own judgment for the judgment of the railroads on the question of whether or not increased rates will yield increased revenue. This construction in effect puts the Commission in the position of business manager of the railroads, passing judgment upon the question of what rates will or will not promote the movement of traffic, which is essentially a matter for determination by those who are responsible for the management of the railroads. The Commission has no direct contact with the subject and can reach its conclusions only on the opinions expressed by witnesses before it; in many instances these witnesses represent the people who would be required to pay the increased rates. The railroad managements would not propose rates which they thought would appreciably reduce the volume of traffic on the railroads and, even if they should make such a mistake, they are the ones who must bear the responsibility for the results of railroad operation. The law should be amended so as to restore the 1920 direction to the Commission to regulate rates in such a manner as to yield a fair return upon railroad property devoted to the public service, and to eliminate the existing admonition that the Commission consider the effect of rates on the volume of traffic.

A number of other proposals have been advanced for increasing the earnings of the railroads legitimately, such as repeal of the so-called "land grants" which now give the Federal Government practically 50% reduction on a large volume of traffic, and repeal of the "long-and-short haul clause" of the Interstate Commerce Act. These, and other proposals, would be of substantial assistance to the railroads in their present struggle with unregulated and subsidized competition; but the most important single legislative step which should be taken at once is the restoration of the 1920 rate-making rule, which would give the railroads a chance to exercise their best judgment to increase their net railway operating income by such rates as the Commission might approve as reasonable.

Increased earnings have a direct bearing on increased operating efficiency. To take advantage of the economies offered by new devices and improvements it is necessary to attract capital, and that depends upon ability to pay interest. The rate of retirement of old equipment is to some extent dependent on the depreciation charges which earnings will stand. The railroads must earn more money before they can eliminate many unnecessary expenditures of this type.

Another class of unnecessary expenditures, of an unproductive character, is for separation of highway grade crossings, for the principal benefit of traffic on the highways; increasing the size of railroad bridge openings, for the benefit of traffic on waterways, and special assessments for municipal street paving, drainage, and other local improvements. Mention of such enforced contributions for structures and improvements of a local nature leads to the thought of the vast sums of money being exacted each year from the railroads for Federal, State, county, city, town, village, school, park, library, and other purposes. Railroad taxation is too large a subject for this kind of discussion. Of course, the railroads should contribute their fair proportion of the cost of Government,

but it is fundamental that the burdens of taxation should be distributed equally. One striking inequality is the heavy taxes on railroad terminal facilities in contrast to tax free municipal barge terminals and airports.

Another cause of unnecessary expenditure is found in the rules and regulations respecting rates of pay and working conditions in railway service. The agreements with the railway labor organizations have not been adapted to changed conditions, and their application results in some instances in exorbitant payments and, in others, payments far greater than the industry can afford. "Make-work" laws, limiting length of trains, size of crews, etc., are of like character.

One of the few railroad questions upon which the writer seems to differ with Mr. Lavis is that of consolidation; perhaps the two views are not so divergent. Mr. Lavis is opposed to consolidation when carried to the extent of building up such large organizations that they lose the sense of personal management, and he expresses approval of reasonable competition in the railway field. With these general expressions the writer is in full agreement, although there might be a difference of opinion as to the precise number of consolidated systems. In England a little more than 20 000 miles of railroads were consolidated into four systems. In the United States approximately 240 000 miles of road were disposed of in eighteen systems in the final consolidation plan of the Commission. Reducing the number of railroads to about eighteen or twenty would eliminate a tremendous amount of over-head organization, would enable traffic to be concentrated on the most favorable routes, using the best parts of the several lines as they now exist, and would automatically bring about the most desirable type of co-ordination of terminals—that is, co-ordination in a few strong ownerships. The beneficial features of competition would be preserved and enhanced, because a relatively few strong roads, able and willing to give good service, would insure a higher quality of competition than could be obtained from too many lines, competing with each other and weakening each other by excessive competition, and unable to use their facilities to capacity. Capital costs would be reduced. All of the indispensable advantages of private operation would be preserved.

Of course, large-scale consolidations would reduce the number of railroad officers and employees. In England, provisions were made for their protection, and the railroads in this country now have the so-called Washington Agreement with the railroad Brotherhoods providing for payment of a "dismissal wage" in case of consolidations. Undoubtedly, some reasonable provision could be made to avoid turning any man out on the street. Although the cost of such provision might postpone for a few years the full economies to be derived from consolidation, the problem is not so large as might be imagined. Mr. Leslie Craven states²⁴ that if the railroads were consolidated into seven or eight major systems, it is probable that about 75 000 men would lose their jobs; whereas, from July, 1937, to June, 1938, railway employment decreased from 1 174 000 to 914 000, or by 260 000 men.

²⁴ "Railroads Under Pressure," by Leslie Craven, *Atlantic Monthly*, November, 1938.

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DISCUSSIONS

SIPHONS AS WATER-LEVEL REGULATORS

Discussion

BY MESSRS. A. GRIFFIN, AND H. P. CURRIN AND D. M. UMPHREY

A. GRIFFIN,⁶ M. AM. SOC. C. E. (by letter).^{6a}—Siphon regulators can be designed and built in many different ways and still perform their intended functions usefully. However, much remains to be learned as to the various factors affecting efficient design and most useful adaptation, and the author's paper is a valuable contribution to the subject.

It has been a debated subject whether a head greater than one atmosphere can be used effectively in a siphon regulator. W. P. Creager,⁷ M. Am. Soc. C. E., has given a mathematical analysis to show that this is possible. The author indicates the same conclusion but does not specifically state it. His tests show that the siphon heads of nearly 46 ft are effectively used in the Walterville siphons. They closely approach the limit of head at Point 4. With a greater extension of the lower leg there would probably be a break in the water column near Point 4. With a shortening of the lower leg to give a head of one atmosphere, or less, there would be a reduction in coefficients of flow and siphon efficiency. The additional head greater than one atmosphere that can be used effectively is related to the energy losses and velocity changes below any point in the outlet leg selected for analysis.

The study of siphon behavior is important to the improvement of design and particularly to confident predetermination of capacity and performance. Violently destructive effects are not likely to be encountered, and there is usually ample opportunity to repair minor injury such as results from cavitation and wear. Nevertheless, it is desirable to reduce the destructive effects of turbulence and high velocities, and to gain the advantages of improved discharge efficiencies. This is especially true where large, or critical, heads are utilized in large-capacity siphons.

In any particular design siphon efficiency may properly be sacrificed in favor of some one or more desired characteristics, or to accommodate construc-

NOTE.—This paper by J. C. Stevens, M. Am. Soc. C. E., was published in October, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

⁶ Chf. Engr., Dept. of Natural Resources, C. P. Ry., Calgary, Alberta, Canada.

^{6a} Received by the Secretary December 12, 1938.

⁷ *Transactions*, Am. Soc. C. E., Vol. 85 (1922), p. 1122.

tion methods. Coefficient of flow and sensitivity or other desired characteristics are likely to be important points in the mind of a designer. Coefficients of flow will tend to be greater for smaller heads. Keeping in mind, also, that velocity varies as the square root of the head, it is evident that siphons utilizing comparatively low heads (even as low as 1 ft or 2 ft) may be employed usefully. A siphon with a head of 6 ft to 8 ft will probably discharge half as much as one with the maximum usable head.

A siphon is not likely to be clogged by any quantity of drift in pieces short enough to pass through it readily. The writer has seen a 4-ft by 4-ft siphon passing tangled masses of Russian thistles at an almost unbelievable rate, and has never seen a siphon clog. Two adjacent siphons, operating simultaneously, would be more likely to clog because of drift catching on the partition wall between them. This is a source of risk in any multiple-opening structure that is passing drift. It is very difficult to clog a single-opening structure.

Perhaps the greatest and most common service that can be performed by a siphon regulator is as a strictly automatic regulator in which rôle it is almost perfect. For such use the writer suggests that it is most desirable to eliminate every form of manual control or adjustment. Of course, a siphon can also perform services in which manual control or adjustment is included as an essential feature.

It is desirable to emphasize to engineers the ease of design and construction and the wide variety of useful applications of the siphon regulator. In one case the writer built two inside forms, fixed the reinforcing in place, and then applied gunite. The completed siphons were hauled to location and hung in place over the top of a previously constructed concrete wall. The summit vertical dimension was 6 in., the width 3 ft, and the head about 9 ft. One siphon was given a sealing basin, the other discharged freely, and both gave the same satisfactory service. In 1938 a siphon of about 30 cu ft per sec capacity, utilizing about 6 ft of head, was welded together in the shop in sheet metal. It was installed by hanging it over a flash board in an irrigation-canal structure where it performs a very useful service. It is also hung to a screw lift so that the control elevation can be altered.

For most cases the siphon is effective from zero discharge to 100% of its capacity. When the flow is too small for the siphon to operate continuously it can still operate as an overflow spillway and intermittently as a siphon.

The writer likes the author's designation "siphon regulator." This is more adequately descriptive of its field of usefulness than the more limited term "siphon spillway." He disagrees with the term "primer." This opening is an air inlet to break siphonic action and thus would be better described as a "de-primer," or simply as an "air-inlet." At times it would be useful to have more information than is now available on the factors determining the minimum dimensions of the air-inlet necessary to interrupt siphonic action. It would be interesting to know whether any tests were made of the capped pipes at the summits, with the air-inlets ("primers") sealed.

The writer has never used a nappe aerator and would be interested in comments by the author as to conditions under which it is either necessary or desirable.

Engineers interested in siphon regulators should re-read the paper and discussions on this subject by the late George F. Stickney,⁸ M. Am. Soc. C. E., published in 1922, and that by the author⁹ published in 1934.

H. P. CURRIN,¹⁰ Esq., AND D. M. UMPHREY,¹¹ Esq. (by letter).^{11a}—Experience with the Leaburg siphons mentioned by the author indicated that these siphons would maintain flow down to about 25% of their capacity before breaking. At Leaburg there are five siphons rated at 500 cu ft per sec each, one rated at 250 cu ft per sec, and one at 100 cu ft per sec. It was decided that the Walterville siphons could be designed with fewer units of higher capacity. This would reduce the cost materially. A study was made at Leaburg¹² to determine the factors favoring the lower partial flows so as to take advantage of these points in the Walterville design. The backward sloping vertical legs at Leaburg are about 28 ft long. The lower leg is terminated about 25 ft from the end of the sealing pool section as contrasted with about 68 ft finally adopted for the Walterville installation.

With the Leaburg siphons running at full flow the water completely fills the outlet. With a small air admission at the primer the water falls away from the top of the outlet, venting back up the barrel and thus reducing the head on the siphon. The Walterville siphons were designed with a long lower leg to take advantage of this condition. Fig. 8 shows that this feature was justified. The shorter length above the sealing pool allows the siphon to prime faster, as less air must be removed to start siphonic action. These design features were favored by the configuration of the forebay wall and lower hillside at Walterville.

At Leaburg air is admitted under the primer into a narrow space. From that point it is drawn upward into a horizontal slot and discharged into the highest point of the upper crown where there is a very low absolute pressure when the siphon is running full and pressure is still considerably below that of the atmosphere at low partial flows. This results in a high velocity through the constricted passage behind the primer. Water is drawn several inches above the surface of the forebay into this passage, and the hydraulic conditions are such that there is an alternate passage of water and air through the priming passage in a series of gulps. When the siphon is running nearly full these gulps occur several times a second; but as the forebay falls and more air is admitted the period lengthens until the gulps occur about once a second when the siphon finally breaks at about 25% flow.

It was felt that this surging action had a tendency to break the siphon sooner than if the air were admitted steadily, and special attention was paid to eliminating this surge at Walterville, as far as possible. Fig. 1 shows how this was done. Air is admitted into each barrel as directly as possible from just

⁸ *Transactions, Am. Soc. C. E.*, Vol. 85 (1922), p. 1098.

⁹ *Transactions, Am. Soc. C. E.*, Vol. 99 (1934), p. 988.

¹⁰ Supt., Elec. Dept., Eugene Water Board, Eugene, Ore.

¹¹ Asst. Elec. Engr., Eugene Water Board, Eugene, Ore.

^{11a} Received by the Secretary December 10, 1938.

¹² For illustrations, see *Transactions, Am. Soc. C. E.*, Vol. 99 (1934), p. 989.

above the forebay level. Although it was not possible to do away with the gulps entirely, they occur more frequently and are less violent than at Leaburg.

The horizontal primer lips shown in Fig. 1(a) were installed after the siphons had been in service long enough to show how far the surface drop-down curve, mentioned by the author, extended from the primer opening. Later these 12 in. lips were added.

Observations before and after adding the lips indicated that these devices caused Siphon No. 1 to run full 0.27 ft, and Siphon No. 2 to run 0.23 ft, below the forebay level necessary to shut off regulating air without them. Breaking and priming levels were affected very slightly, if at all.

An important element in the regulation of these siphons to low flows is the difference in elevation between the upper crest and the priming air intake. In each of the Walterville siphons, the top edge of the intake opening is set 0.2 ft above the corresponding crest.

In Table 2 it is shown that the lowest measured flows in both barrels occurred with the forebay at approximately 0.2 ft above the corresponding crest or at about the level of the lower edge of the priming lip. Although this head over the crest is not sufficient to re-establish siphonic action after the siphon has broken, it does tend to stabilize the lower flows which otherwise might be interrupted by surging in the siphon barrels. When these openings were lowered below their present position for testing the siphons were less stable at the lower flows and broke at a higher minimum discharge.

A 12-in. pipe was installed as a vent in the summit of the lower sealing curve in Siphon No. 1 and a 10-in. pipe in Siphon No. 2. It was thought that there might be some tendency of the water to cling to the tops of the long lower sections of the barrels at lower flows, and make it necessary to admit air through these vents by means of auxiliary regulating hoods in the forebay to vent the lower sections. Experience showed that this was not necessary, and the vents were closed by bolted flanges.

There has been some trouble with ice in the primers of the Leaburg siphons.¹³ The high velocity and consequent sharp reduction of temperature of the admitted air in the constricted passages caused the primers to choke up with ice in a place very difficult to reach for clearing. Finally, 6-in. iron pipe nipples were cemented into holes drilled into the crowns of these siphons so that air could be admitted to break the siphons or to regulate their flow when the primers were closed by ice. This feature was also used at Walterville.

In the winter of 1936-1937, ice occurred at both Walterville and Leaburg. At Walterville some ice formed beneath the primer lips on both siphons, but it was easily removed. In a locality where ice is common it would probably be advantageous to install some heating device on the primer lips. These venting openings were very convenient while testing.

The author gives the priming level for both siphons with the forebay rising slowly. In 1936 a test was made to determine the effect, on the priming level, of raising the water quickly. The plant was shut down as fast as possible under normal operation. The forebay water raised at the rate of 0.38 ft per

¹³ *Transactions, Am. Soc. C. E.*, Vol. 99 (1934), p. 1011.

min. Both siphons primed together at Elevation 595.6 which is 0.2 ft above the normal priming level for Siphon No. 1, and 0.37 ft for Siphon No. 2.

In the Walterville design special care was taken to avoid air leakage around the adjustable primers. Smooth steel faces were used for all sliding joints, and jack bolts were provided to hold these joints together. It was found necessary to calk these joints to make them tight. Some form of gasket should have been included in this design.

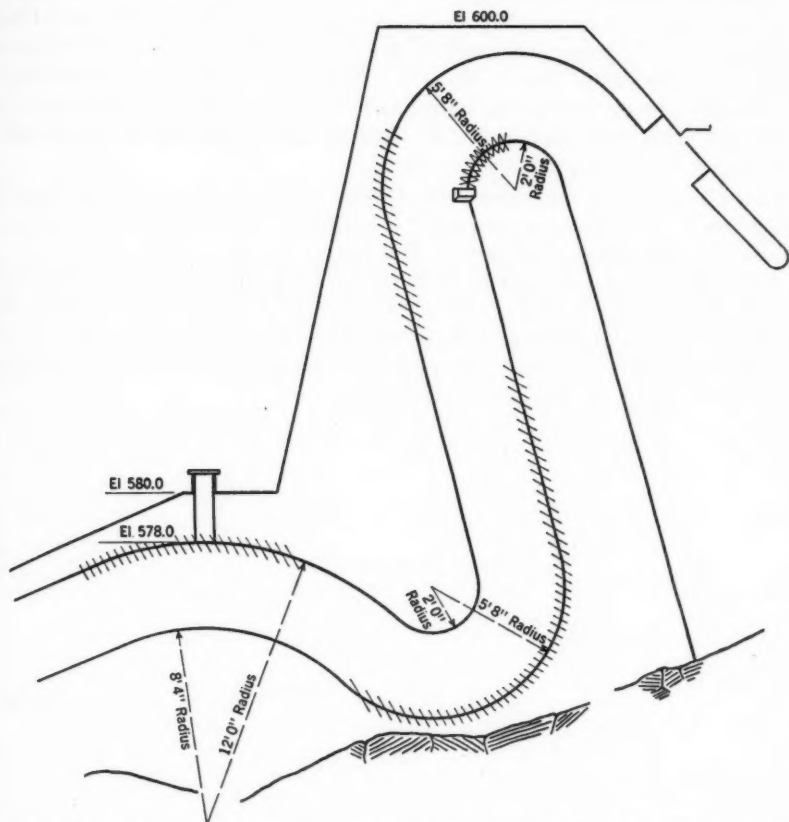


FIG. 9.—WEAR IN THE WALTERVILLE (ORE.) SIPHON SPILLWAY AS SHOWN BY INSPECTION OF NOVEMBER 22, 1938 (SINGLE HATCHING INDICATES POINTS OF WEAR ON BOTH SIPHONS; AND DOUBLE HATCHING INDICATES WEAR IN SIPHON NO. 2 ONLY)

The Walterville siphons vibrate to a certain extent at full flows. This is similar to the effect at Leaburg mentioned by the author¹⁴ and is probably due to the same cause; that is, the water column separates and rejoins in the upper part of the vertical section.

Inspection of the Walterville siphons late in 1938 showed some slight erosion on the outside of the curves where water velocity is highest. Some erosion had

¹⁴ *Transactions, Am. Soc. C. E.*, Vol. 99 (1934), p. 998.

also occurred in Siphon No. 2 on the inside of the upper curve, extending downward to the nappe aerator, or from Section 2 to Section 3 (see Fig. 9).

The wear from Section 2 to Section 3 in Siphon No. 2 is in a place where absolute pressure is very low when the siphon is discharging the higher flow (see Fig. 7). This is also at a location where it was found very difficult to place concrete properly. A gravel pocket was found in this area after the inside form was removed during construction, and was filled by a surface patch. This patch shows very little wear.

Under normal operation Siphon No. 2 runs continuously with partial flow, and Siphon No. 1 primes only after Siphon No. 2 has built up a considerable discharge. Evidently, Siphon No. 2 is subjected to more severe conditions of low absolute pressure inside the barrel and should show more deterioration due to this condition than Siphon No. 1. The erosion in Siphon No. 2, Section 2 to Section 3, is no doubt due to cavitation.

Low absolute pressure occurs at all four piezometers at Section 4 at the higher flows. This section is in good condition in both siphons. Dense concrete is essential under conditions of this nature.

After eight years of experience with the Leaburg siphons and two years with those at Walterville, the writers feel that they would not hesitate to build other siphons for a similar purpose. It is felt that they are superior to any mechanical gate for this service. To quote from a report from the man in charge of operation "they are 'darn' good spillways."

ANALYSIS OF RUN-OFF CHARACTERISTICS

Discussion

BY MESSRS. VICTOR H. COCHRANE, AND BERTRAM S. BARNES

VICTOR H. COCHRANE,⁹ M. AM. Soc. C. E. (by letter).^{9a}—The graph which the author refers to as a "histo-gram" is very useful in the study of flood flows. That kind of histogram in which the abscissas express the time required for the water to flow to the point of measurement may be termed the time-area diagram. Its integral becomes the concentration curve, or ascending limb of the hydrograph, as stated by the author. It may also be employed in a simple manner to determine the ordinates of the descending limb of the hydrograph.

As an illustration, consider the simple case shown in Fig. 8. The watershed (Fig. 8(a)) is divided into six areas, or zones, such that the run-off there-

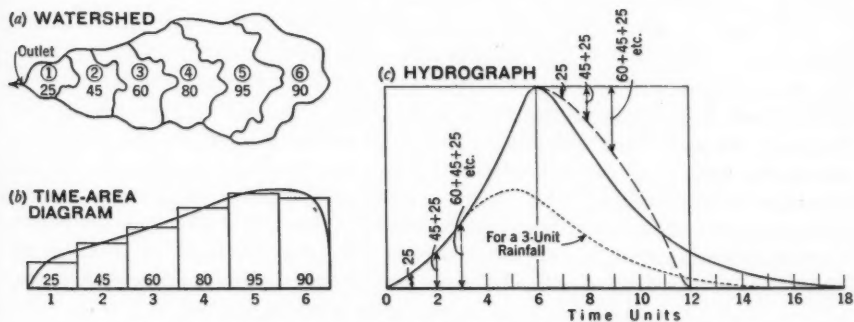


FIG. 8.—WATERSHED, TIME-AREA DIAGRAM, AND HYDROGRAPH

from, due to a rainfall of uniform intensity, arrives at the outlet in equal time intervals. The corresponding time-area diagram is shown in Fig. 8(b), and the resulting hydrograph in Fig. 8(c). The ordinates for the concentration curve are computed by summing the zone areas, as shown in Table 5. If the velocities during the falling stage were the same as during the concentration

NOTE.—This paper by Otto H. Meyer, Assoc. M. Am. Soc. C. E., was published in November, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

⁹ Cons. Engr., Tulsa, Okla.

^{9a} Received by the Secretary December 23, 1938.

period, the ordinates of the descending limb of the hydrograph would be obtained directly by deducting from the peak flow the various zone contributions in succession, as in Table 5. The resulting curve is shown in the broken line, Fig. 8(c). This curve, plotted downward with reference to the horizontal line representing the peak flow, is the same as the concentration curve. It will be seen that if the ascending limb of the hydrograph rises slowly in the early stages, due to the narrowness of the water-shed or the flatness of the slopes near the outlet, the hydrograph tends to fall slowly after the peak, and *vice versa*.

TABLE 5.—COMPUTATION FOR THE ORDINATES OF A HYDROGRAPH

Item No.	Zone No.	ORDINATE AT THE END OF EACH TIME-UNIT NUMBER:											
		1	2	3	4	5	6	7	8	9	10	11	12
1	1	25	25	25	25	25	25						
2	2		45	45	45	45	45	45					
3	3			60	60	60	60	60	60				
4	4				80	80	80	80	80	80			
5	5					95	95	95	95	95	95		
6	6						90	90	90	90	90	90	0
7	Totals	25	70	130	210	305	395	370	325	265	185	90	0
8	Average ordinate of descending limb.....						383	348	295	225	137	45	
9	Relative times of flow.....						1.01	1.11	1.37	1.87	2.70	3.94	
10	Discharge in unit time (Item 8 ÷ Item 9).....						379	314	215	120	51	11.5	
11	Corresponding abscissa; unit times*.....						6.50	7.56	8.80	10.42	12.71	16.03	

* Base of hydrograph is 18 time units.

The base of the falling limb of the hydrograph must exceed that of the rising limb. That this is true will be seen upon consideration of the flow from the zone farthest up stream. The initial run-off from this zone rides the crest of the flood, as it were, during the concentration period, thus traveling to the outlet at the maximum velocity. The final contribution flows at, or a little above, the base stage, with a relatively low velocity. The base of the hydrograph for falling stages is the time required for the water to travel from the most remote zone to the outlet at the ordinary or low-water stage of the stream. For practical purposes, a somewhat shorter time may be used, as the flow at final stages is relatively small. The base flow definitely limits the length of the flood period. Where the author states (following Equation (3)) that "the length of base is actually infinite," he is referring no doubt to the mathematical characteristics of his formula.

After the peak flow is reached, the velocity, which varies approximately as the square root of the hydraulic radius, decreases quite slowly at first; then much more rapidly during the last stages of the flood. The rate of change in velocity with respect to time can be determined approximately from the characteristics of the stream channel. As the time period corresponding to a given element of flow is lengthened, the discharge in unit time is decreased in inverse ratio. In Table 5 the increase in length-of-flow periods is assumed to vary as the cube of the number of units of flow (six in this case), and the base

of the descending limb is twice that of the ascending limb. The resulting curve is shown in Fig. 8(c).

For a rainfall extending over a shorter period than that producing the maximum flow, the summations and adjustments are made in a similar manner. In Table 5 the summations would include as many terms in each horizontal line as there are periods of precipitation. The dotted line in Fig. 8(c) shows the hydrograph for a rainfall extending over three time units.

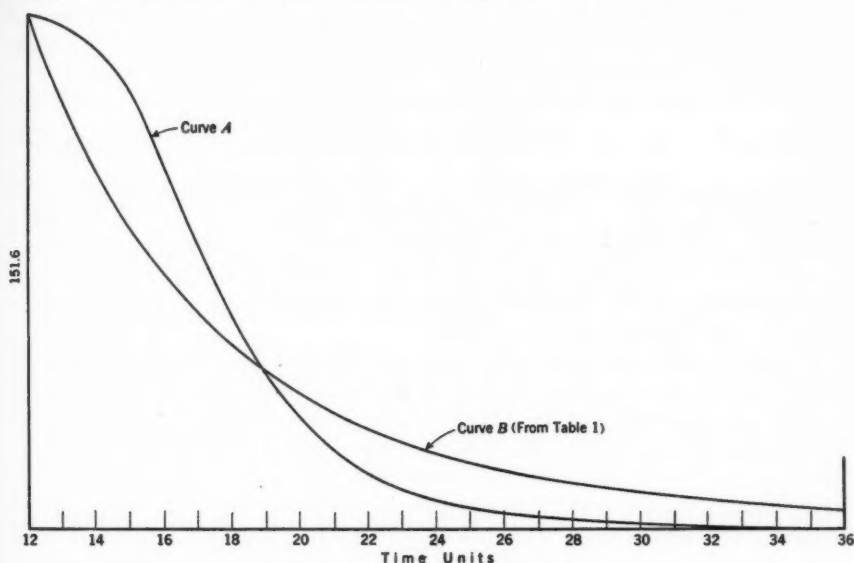


FIG. 9.—COMPARATIVE HYDROGRAPHS

The author's unqualified assertion that there is a sharp break in the basic hydrograph is therefore open to question. Such a break might be observed in the case of a stream with a wide fan-shaped area at the down-stream end and a long narrow valley farther up stream. In a water-shed of this type the maximum flow might come from a rainfall covering only the lower part of the drainage area, the concentration period being shorter, and the intensity of precipitation greater, than for the full area. The author's logarithmic formula gives ordinates too small for the upper part of the curve and too large for the lower part. In Fig. 9, Curve A shows the falling limb constructed by the method just described, assuming a base length double that of the concentration curve, and Curve B is derived from Table 1.

For a given uniform intensity of precipitation and a constant run-off rate, the peak flow per unit of area is the same regardless of the size or shape of the water-shed. The pattern of the drainage area becomes a factor in case the rainfall is of variable intensity with respect to time or area. Inasmuch as the average intensity diminishes rapidly with increase of time, the time of concentration (the length of stream divided by the average velocity) is a much more reliable indicator of maximum discharge than either area or length of water-

shed. The writer has heretofore proposed formulas¹⁰ for maximum discharge from small water-sheds, as follows:

$$q = c I (A + B) \dots \dots \dots (19)$$

in which, for $T = 1$ hr or less,

$$A = 645 \frac{6 + I}{6 T + I} \dots \dots \dots (20a)$$

or, for T greater than 1 hr,

$$A = \frac{645}{T^{0.75}} \dots \dots \dots (20b)$$

and

$$B = 100 (z - 1) \frac{6\sqrt{T} - T}{0.4 I + T} \dots \dots \dots (21)$$

In Equations (19), (20), and (21): q = flood discharge in cu ft per sec per sq mile of drainage area; c = the ratio of run-off to rainfall; I = average intensity of maximum precipitation for the aggregate period of 1 hr during a given storm; T = time of concentration in hours; and z = ratio of maximum to average area of divisions of the time-area diagram (or, roughly, the ratio of maximum width of water-shed to the mean width).

It will be seen that these expressions are designed to improve the rational formula in two respects: The intensity of precipitation is made to vary with the time of concentration, and the effect of non-uniform precipitation is included. In Equation (19) the term A expresses the flow due to a uniform rate of rainfall, whereas the term B gives the additional flow due to the most extreme non-uniform precipitation. With the exception of the run-off coefficient, c , these equations contain no terms with widely varying and uncertain values, such as appear in many other run-off formulas. The time-intensity relationship in Equation (20a) has been shown¹¹ to agree well with actual rainfall records. The empirical expression for B in Equation (21) represents values computed from typical time-area diagrams, assuming that the maximum run-off occurs

TABLE 6.—VALUES OF q FOR: $I = 3.0$; $c = 0.5$; AND $z = 1.5$

Description	VALUES CORRESPONDING TO TIME OF CONCENTRATION, T , IN HOURS						
	0	0.25	0.50	1	6	12	24
Average intensity, I , in inches.....	9	6	4.5	3	0.78	0.465	0.276
Substitution Factors:							
A (Equations (20)).....	1 935	1 290	967	645	168	100	59.5
B (Equation (21)).....	0	95	110	113	61	33	10.5
q (Equation (19)).....	2 900	2 080	1 615	1 137	344	200	105

when the contribution from the maximum area, due to the most intense precipitation, arrives at the outlet.

These formulas may be useful in estimating peak flows for water-sheds of different sizes. They cover the entire range of possible values. The four

¹⁰ *Engineering News-Record*, November 25, 1937, p. 864.

¹¹ *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 369.

variables used, c , I , T , and z , may be evaluated for any particular case by the application of appropriate methods or pertinent data.

Values of q are shown in Table 6 for $I = 3.0$, $c = 0.5$, and $z = 1.5$. It will be noted that q diminishes very rapidly as the time of concentration increases.

In Equation (20b), the exponent of T ($= 0.75$) is an average value, convenient for use with the slide-rule. Other values may be used to suit the rainfall characteristics. The average intensity per hour, i , for a 24-hr period, may be substituted for the term, I , according to the relation, $I = 10.9 i$.

The foregoing formulas should be of use in transposing basic hydrographs, or in checking results obtained by other methods. They afford a clear picture of the effect of variations in the principal factors involved in flood discharge.

BERTRAM S. BARNES,¹² ASSOC. M. AM. SOC. C. E. (by letter).^{12a}—The new and original attack upon the problem of constructing discharge hydrographs of run-off from observed rainfall, as presented by Mr. Meyer, will arouse considerable interest and will give a new impetus to research in this field. The author has admirably reconciled the apparently opposite viewpoints of the advocates of the unit hydrograph and of the depletion curve.

The assumption that run-off is proportional to rainfall will give much more satisfactory results in hilly country, where a relatively large part of the rainfall enters the stream channels at once, than it will in a flat country where most of the rain goes into the ground. Under the latter conditions, fluctuations in the momentary intensity of rainfall may have a pronounced effect upon the amount of run-off. The run-off factor is one of those devices used for lack of anything better. Its use is made necessary in most cases by the imperfect state of the present-day knowledge of the relation between rainfall and run-off, and because of the absence of sufficient basic data. Hydrologists should not lose sight of the fact that the idea of a run-off factor is a poor concept at best, and that its use is likely to produce very unsatisfactory results wherever the infiltration is large and rainfall varies greatly in momentary intensity. It is unquestionably true that the average monthly or weekly value of the run-off factor rises and falls with the ground-water level. In some cases it even appears for a time to maintain a fixed relation to the rate of ground-water flow, although this must be considered pure coincidence. In the Mississippi Valley the variation of the run-off factor from day to day is frequently so great that an average value, or one predicted from the calendar, has been found to be of little use when the unit hydrograph method was employed. There is urgent need for the development of better methods of predicting the quantity of run-off from a given storm.

Another fact frequently overlooked in present-day studies is that the time of concentration, as it is usually defined, is not necessarily identical with the elapsed time from the occurrence of rainfall to the peak of the run-off. In basins of irregular shape there will almost always be flow from certain tributaries which will still be increasing after the peak of discharge from the basin has passed.

¹² Hydrologic Supervisor, Upper Mississippi Region, U. S. Weather Bureau, Iowa City, Iowa.

^{12a} Received by the Secretary January 3, 1939.

The assumption that, after the storm water has ceased to flow from the surface of the ground, the streams have no source of replenishment other than ground-water flow is one that should be given more critical study, or at least a definite interpretation. Ordinarily, ground-water flow is thought of as the discharge from water held in storage at, or below, the ground-water table. Two or three weeks may sometimes elapse between a rainstorm and the increase in ground-water flow which results from it. Many streams in the Mississippi Valley receive storm water from two immediate sources: (1) The drainage from the ground surface into the stream channels during and immediately after the rain; and (2) what appears to be a lateral movement of percolating water in the upper soil layers, entering the stream channels in the form of seepage, and reaching its maximum flow about 2 days after the peak of storm flow. The latter phenomenon was noted by the writer when making some studies of Zumbro River, in Minnesota, in 1936, and was given the name "secondary base flow." The same, or something quite similar, was observed by Mr. C. R. Hursh¹³ and called by him "subsurface storm flow."

Separation of the storm run-off into surface flow and secondary base flow has been made for a number of streams in the Mississippi Valley. In order to obtain satisfactory results it was necessary to determine the discharge at intervals of 6 hr or less, rather than to use daily averages. It has been found that the recession of each of these components of flow can be plotted as a straight line on semi-logarithmic paper. The formula used in the office of the writer is

$$Q_t = Q_0 K^t \dots \dots \dots (22)$$

in which Q_0 is the discharge at any instant; Q_t is the discharge t days later; and K is known as the "daily depletion factor," its value for secondary base flow being much higher than for surface flow.

Equation (22) can be derived from that proposed by W. W. Horner, M. Am. Soc. C. E., and F. L. Flynt, Assoc. M. Am. Soc. C. E.¹⁴ It should be noted that flow from only one source (surface) was used in developing the Horner and Flynt formula.

In the case of the Iowa River at Marshalltown, Iowa, for a concrete example, it has been found that the secondary base flow becomes appreciable in about 12 hr after the peak of storm discharge has passed and reaches a maximum in from 60 to 65 hr after the peak of storm discharge. At that time it may become as much as four times the surface discharge, or 80% of the total storm discharge. The peak of the hydrograph of secondary base flow has a fairly flat top of about 40-hr duration, compared with about 10 hr for the peak of surface flow, and a depletion factor, K , equal to 0.69, as compared with 0.33 for surface flow. The ratio of the total secondary base run-off to the total storm run-off varies quite widely. For gentle rains the peak discharge of secondary base flow at Marshalltown is about 20% to 25% of the peak discharge of surface flow. For rains of high momentary intensity the ratio may be considerably smaller.

¹³ *Transactions, American Geophysical Union*, 1936, p. 302.

¹⁴ *Transactions, Am. Soc. C. E.*, 1936, Equation (1), p. 151.

Studies, made in the writer's office, of the flow of the Iowa River at Marshalltown, and that of certain other streams, show that the recession of the ground-water flow can be expressed by Equation (22) during periods when no recharge is taking place (the value of K at Marshalltown for ground-water flow is 0.98). It follows, therefore, that for each of the components of flow the quantity of water remaining in live storage at any time during a true recession is directly proportional to the discharge of the component. This reasoning cannot be applied strictly to any flow that includes discharge from more than one of the three sources previously described.

Correction for *Transactions*: In the denominator of Equation (2), $t - t_i$ is the exponent of K ; and, in Equation (9) " ea " should read " e^α ."

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DISCUSSIONS

SPECIFICATION AND DESIGN OF STEEL GUSSET-PLATES

Discussion

BY MESSRS. R. H. SHERLOCK, AND L. E. GRINTER

R. H. SHERLOCK,¹⁷ M. AM. SOC. C. E. (by letter).^{17a}—The gusset-plate is one of the most common details in fabricated steel structures. The intensity and distribution of the stresses in the plates are seldom given any consideration since experience has shown that in ordinary cases a plate will not fail if it is big enough to contain the required number of rivets and has such a shape that it can be cut and fabricated economically. Unusual plates are sometimes analyzed according to the elementary principles of mechanics, one standard specification requiring that "The gusset plates shall be of ample thickness to resist shear, direct stress, flexure, etc."

There is relatively little published material, either theoretical or experimental, dealing with the stress distribution in gusset-plates, so that the engineer finds little to support his judgment in the design of joints lying well outside of previous experience. The stress trajectories in a plate have been found by the mathematical theory of elasticity for only two boundary conditions. In both cases, the load is applied through a single rivet—in the one case to a plate of infinite extent, and in the other case to one-half of an infinite plate bounded by a straight line passing through the rivet. The laboratory research worker knows in advance that there will be little if any opportunity to embellish his report with an elegant comparison between his test results and the results of rational analysis. Progress in this field then must consist in the accumulation of experimental results from a large variety of plate shapes and conditions of loading, thus extending the basis upon which the engineer may exercise his judgment in design.

The author has added to the knowledge in this field through the careful refinement and application of some of the newer laboratory techniques, and through a discussion of present practice in the light of his own results and

NOTE.—This paper by T. H. Rust, Assoc. M. Am. Soc. C. E., was published in November, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

¹⁷ Prof. of Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

^{17a} Received by the Secretary December 28, 1938.

those of other investigators. It is hoped that the same technique may be applied to more complicated shapes of plates and conditions of loading.

An interesting feature of the photographs in Fig. 3, not referred to by the author, appears in Models B_1 , B_5 , C_4' and C_5' . Here, the fringes, caused by the light passing through the members attached to the gusset-plates, are inclined to the axis of the members, thus indicating a non-uniform distribution of the stress on the cross-sections of the members. This is in sharp contrast to the fringes in the symmetrical Model C_1' where the fringes indicate a uniform distribution of stress across the members. This indicates again that, if the same welding or riveting is supplied on both sides of the member, the greater stress will occur in the welds attached to the side having the larger area of gusset-plate. All of this is merely another way of showing that the flow of stress is along the path of greatest rigidity.

L. E. GRINTER,¹⁸ M. AM. SOC. C. E. (by letter).^{18a}—It seems strange at first thought that so much attention has been paid to the design of truss members for direct stresses and secondary stresses, whereas relatively little attention has been given to the analysis or design of gusset-plates. The explanation, of course, is that stress analysis of a gusset-plate is a complex process dependent upon the mathematical theory of elasticity, and, as the author has stated, only the simpler kinds of gussets have been subject to analysis at all. It is commendable, therefore, that Mr. Rust has found a device for representing, with reasonable assurance, the relative stress patterns in joints of various shapes and sizes. The meticulous care with which the author has prepared his models, which was observed by the writer, leads to more than usual confidence in the usefulness of the results. His use of the light interferometer for determining the sum of the principal stresses is highly commendable.

One feature of gusset design that seems to be neglected more often than it is considered is the simple one of flexure. There is nothing new about the requirement that the gussets of Fig. 18 must be designed for a combination of

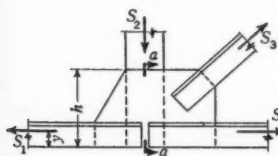


FIG. 18.—ECCENTRICITY AND GUSSET FLEXURE

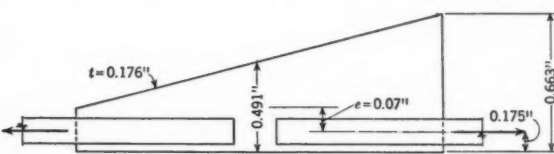


FIG. 19.—ECCENTRICITY OF MODEL B_4

direct stress and flexure, but it is often misunderstood. Since the total stress, S_1 , must be transferred across the broken lower chord member by the gusset-plate, it is evident that there is flexure to be resisted along Section a-a. The flexural moment is,

$$M = S_1 \left(\frac{h}{2} - y \right) \dots \dots \dots (3)$$

¹⁸ Dean, Graduate Div., and Director of Civ. Eng., Armour Inst. of Technology, Chicago, Ill.

^{18a} Received by the Secretary December 8, 1938.

Then the total fiber stress may be estimated as,

$$f = \frac{S_1}{h t} + S_1 \left(\frac{h}{2} - y \right) \frac{6}{t h^2} \dots \dots \dots (4)$$

in which t is the gusset thickness resisting the forces shown in Fig. 18. This type of design, in which the gusset is made to serve the function of a splice plate, is not recommended in any specification but it is still used.

It is possible to check this simple analysis against the results obtained by the author for two unsymmetrical models in Fig. 9. Consider Model B_3 for a load of 140 lb. The value of $p - q$ at Point 7L, which is the mid-point, is 2 450 lb per sq in. The corresponding value of $p + q$ is 2 150. The value of p , therefore, is the average of these, or 2 300 lb per sq in. This stress can be recomputed by Equations (3) and (4): $M = 140 \times 0.07 = 9.8$ in-lb; and,

$$f = \frac{140}{0.491 \times 0.176} + \frac{9.8 \times 6}{0.176 \times 0.491^2} = 3\,000 \text{ lb per sq in.}$$

Thus, for this case, the approximate analysis given by Equations (3) and (4) proves to be safe since it over-estimates the maximum plate stress by 30 per cent. In Model B_4 , at a load of 149.6 lb, the actual observed stress is 1 950 lb per sq in. and the fiber stress from Equation (4) is 2 360 lb per sq in., an excess of 20 per cent. The eccentricity is considerably less important for this model than for the one studied first.

Corrections for *Transactions*: November, 1938, *Proceedings*, p. 1834, Line 3 below Fig. 6 should read: "equal to the positive area (Fig. 6(d) = Fig. 6(a) + Fig. 6(b); and, Fig. 6(e) = Fig. 6(a) + Fig. 6(b) + Fig. 6(c)"; p. 1838, Fig. 10, the joint should be shown in the elevation by two full short lines in the center plate; p. 1842, Line 6, omit the word "Furthermore"; p. 1844, Equation

(2) should read, " $\log c = \frac{I_M}{I_G} \left[\frac{1}{K} \left(\frac{I_G}{I_M} \right)^K - K \right]$ "; and change the following

line to read: "in which $K = \frac{a}{\bar{p}}$; a = distance," etc.